Critical values for 22 discordancy test variants for outliers in normal samples up to sizes 100, and applications in science and engineering

Surendra P. Verma and Alfredo Quiroz-Ruiz

Electronic supplement 23-3-01

Critical values for discordancy tests

Revista Mexicana de Ciencias Geológicas, v. 23, núm. 3, 2006, 302-319

Table A1. Critical values for discordancy test N1 of an upper or lower outlier in a normal sample.

	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
n	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
3		1.0982	1.1295	1.1484	1.1531	1.1544	1.1546	1.1547
4		1.2750	1.3500	1.4249	1.4625	1.4850	1.4925	1.4962
5		1.3946	1.4897	1.6017	1.6714	1.7253	1.7489	1.7637
6		1.4888	1.5945	1.7290	1.8222	1.9036	1.9442	1.9729
7		1.5667	1.6787	1.8282	1.9384	2.0417	2.0972	2.1389
8		1.6331	1.7492	1.9092	2.0317	2.1527	2.2210	2.2744
9		1.6908	1.8099	1.9775	2.1099	2.2446	2.3230	2.3866
10		1.7416	1.8630	2.0362	2.1761	2.3221	2.4097	2.4821
11		1.7865	1.9100	2.0879	2.2340	2.3897	2.4847	2.5639
12		1.8273	1.9527	2.1343	2.2848	2.4478	2.5490	2.6352
13		1.8640	1.9909	2.1756	2.3308	2.5006	2.6078	2.6992
14		1.8977	2.0259	2.2133	2.3718	2.5469	2.6584	2.7553
15		1.9285	2.0579	2.2475	2.4090	2.5890	2.7049	2.8060
16		1.9572	2.0876	2.2793	2.4431	2.6274	2.7467	2.8522
17		1.9837	2.1149	2.3086	2.4751	2.6634	2.7855	2.8938
18		2.0088	2.1408	2.3358	2.5041	2.6957	2.8208	2.9320
19		2.0320	2.1646	2.3611	2.5314	2.7254	2.8533	2.9675
20		2.0539	2.1873	2.3850	2.5565	2.7535	2.8836	3.0003
21		2.0746	2.2085	2.4073	2.5803	2.7796	2.9117	3.0313
22		2.0942	2.2286	2.4285	2.6030	2.8044	2.9384	3.0596
23		2.1128	2.2476	2.4485	2.6241	2.8277	2.9639	3.0866
24		2.1306	2.2658	2.4674	2.6442	2.8489	2.9860	3.1117
25		2.1474	2.2827	2.4853	2.6633	2.8699	3.0091	3.1357
26		2.1634	2.2993	2.5024	2.6809	2.8893	3.0296	3.1574
27		2.1792	2.3151	2.5191	2.6985	2.9076	3.0498	3.1783
28		2.1939	2.3302	2.5342	2.7142	2.9249	3.0676	3.1981
29		2.2083	2.3445	2.5494	2.7301	2.9421	3.0855	3.2169
30		2.2219	2.3585	2.5636	2.7447	2.9580	3.1029	3.2358
31		2.2351	2.3719	2.5775	2.7594	2.9738	3.1194	3.2531
32		2.2477	2.3848	2.5910	2.7730	2.9876	3.1343	3.2689
33		2.2600	2.3972	2.6034	2.7867	3.0027	3.1495	3.2848
34		2.2718	2.4091	2.6159	2.7992	3.0156	3.1637	3.3002
35		2.2831	2.4207	2.6277	2.8115	3.0290	3.1780	3.3150
36		2.2944	2.4320	2.6391	2.8235	3.0412	3.1905	3.3286
37		2.3052	2.4429	2.6503	2.8349	3.0537	3.2036	3.3425
38		2.3155	2.4531	2.6610	2.8461	3.0656	3.2166	3.3546
39		2.3255	2.4634	2.6716	2.8568	3.0769	3.2273	3.3689
40		2.3353	2.4731	2.6815	2.8673	3.0878	3.2403	3.3809
41		2.3449	2.4829	2.6914	2.8770	3.0978	3.2498	3.3923
42		2.3542	2.4921	2.7007	2.8866	3.1082	3.2608	3.4023
43		2.3631	2.5014	2.7101	2.8963	3.1186	3.2721	3.4157
44		2.3720	2.5103	2.7193	2.9056	3.1280	3.2819	3.4251
45		2.3806	2.5189	2.7281	2.9148	3.1376	3.2912	3.4349
46		2.3890	2.5273	2.7368	2.9233	3.1468	3.3010	3.4454
47		2.3970	2.5355	2.7447	2.9320	3.1557	3.3105	3.4557
48		2.4052	2.5435	2.7529	2.9404	3.1644	3.3193	3.4638
49		2.4130	2.5513	2.7609	2.9484	3.1726	3.3281	3.4735
50		2.4205	2.5587	2.7682	2.9562	3.1810	3.3364	3.4812
51		2.4280	2.5664	2.7760	2.9639	3.1885	3.3457	3.4919
52		2.4352	2.5736	2.7834	2.9713	3.1969	3.3527	3.5000
53		2.4424	2.5808	2.7908	2.9786	3.2041	3.3610	3.5085

continues

Table A1 (continued). Critical values for discordancy test N1 of an upper or lower outlier in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
54		2.4492	2.5878	2.7976	2.9856	3.2113	3.3678	3.5157
55		2.4561	2.5944	2.8043	2.9926	3.2190	3.3753	3.5229
56		2.4628	2.6012	2.8111	2.9995	3.2257	3.3830	3.5306
57		2.4694	2.6077	2.8177	3.0063	3.2323	3.3897	3.5370
58		2.4758	2.6143	2.8242	3.0128	3.2393	3.3970	3.5451
59		2.4820	2.6204	2.8303	3.0192	3.2453	3.4039	3.5520
60		2.4881	2.6267	2.8370	3.0255	3.2521	3.4098	3.5589
61		2.4944	2.6327	2.8427	3.0314	3.2588	3.4171	3.5664
62		2.5003	2.6387	2.8489	3.0378	3.2652	3.4236	3.5728
63		2.5062	2.6444	2.8546	3.0438	3.2706	3.4299	3.5791
64		2.5121	2.6504	2.8606	3.0499	3.2762	3.4355	3.5850
65		2.5175	2.6559	2.8663	3.0552	3.2831	3.4420	3.5912
66		2.5231	2.6614	2.8717	3.0612	3.2887	3.4479	3.5973
67		2.5286	2.6670	2.8773	3.0669	3.2947	3.4541	3.6034
68		2.5340	2.6723	2.8824	3.0719	3.2996	3.4586	3.6094
69		2.5390	2.6774	2.8878	3.0768	3.3049	3.4649	3.6155
70		2.5444	2.6827	2.8930	3.0823	3.3107	3.4700	3.6204
71		2.5497	2.6878	2.8982	3.0871	3.3159	3.4760	3.6257
72		2.5546	2.6929	2.9030	3.0923	3.3208	3.4814	3.6319
73		2.5596	2.6978	2.9079	3.0976	3.3257	3.4862	3.6365
74		2.5643	2.7025	2.9127	3.1018	3.3308	3.4912	3.6422
75		2.5692	2.7073	2.9177	3.1069	3.3356	3.4960	3.6462
76		2.5739	2.7119	2.9222	3.1117	3.3407	3.5009	3.6517
77		2.5786	2.7167	2.9267	3.1161	3.3452	3.5056	3.6570
78		2.5832	2.7212	2.9313	3.1206	3.3493	3.5104	3.6615
79		2.5877	2.7256	2.9359	3.1255	3.3545	3.5152	3.6670
80		2.5922	2.7300	2.9404	3.1300	3.3590	3.5204	3.6712
81		2.5966	2.7345	2.9448	3.1344	3.3635	3.5246	3.6767
82		2.6010	2.7389	2.9491	3.1389	3.3680	3.5286	3.6808
83		2.6052	2.7431	2.9532	3.1430	3.3720	3.5332	3.6849
84		2.6095	2.7473	2.9572	3.1469	3.3760	3.5374	3.6897
85		2.6136	2.7515	2.9616	3.1511	3.3807	3.5423	3.6946
86		2.6177	2.7556	2.9657	3.1554	3.3847	3.5461	3.6979
87		2.6217	2.7597	2.9698	3.1593	3.3888	3.5506	3.7036
88		2.6259	2.7637	2.9739	3.1631	3.3931	3.5548	3.7073
89		2.6299	2.7678	2.9777	3.1672	3.3968	3.5584	3.7111
90		2.6337	2.7715	2.9813	3.1709	3.4007	3.5623	3.7148
91		2.6375	2.7752	2.9850	3.1749	3.4052	3.5661	3.7187
92		2.6414	2.7792	2.9890	3.1787	3.4086	3.5698	3.7225
93		2.6450	2.7828	2.9928	3.1827	3.4121	3.5741	3.7266
94		2.6489	2.7865	2.9964	3.1863	3.4161	3.5781	3.7307
95		2.6524	2.7901	3.0000	3.1898	3.4194	3.5816	3.7349
96		2.6562	2.7937	3.0034	3.1931	3.4230	3.5847	3.7387
97		2.6598	2.7974	3.0069	3.1967	3.4268	3.5886	3.7422
98		2.6633	2.8009	3.0106	3.2000	3.4303	3.5921	3.7455
99		2.6667	2.8043	3.0139	3.2031	3.4334	3.5963	3.7498
100		2.6704	2.8077	3.0174	3.2070	3.4374	3.6000	3.7529

CL: Confidence level (%); SL: Significance level (%); α: Significance level. The headers for CL or SL or α generally used for most applications are given in italic-bold face. The mean values of the standard error of the mean ($\bar{x}_{se}$) for these critical values ($\bar{x}$) are (respective % errors are also reported in parentheses): ~0.0006 (for α=0.30, 0.040%); ~0.0006 (for α=0.20, 0.041%); ~0.0007 (for α=0.10, 0.042%); ~0.0008 (for α=0.05, 0.043%); ~0.0009 (for α=0.02, 0.045%); ~0.0010 (for α=0.01, 0.05%); and ~0.0011 (for α=0.005, 0.05%).

Table A2. Critical values for discordancy test N2 of an extreme outlier in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
3		1.1405	1.1484	1.1531	1.1543	1.1546	1.1547	1.1547
4		1.3875	1.4250	1.4625	1.4812	1.4925	1.4962	1.4981
5		1.5428	1.6018	1.6715	1.7150	1.7489	1.7637	1.7731
6		1.6565	1.7291	1.8224	1.8871	1.9443	1.9729	1.9932
7		1.7464	1.8281	1.9385	2.0202	2.0974	2.1389	2.1704
8		1.8208	1.9092	2.0318	2.1268	2.2212	2.2746	2.3167
9		1.8840	1.9775	2.1098	2.2151	2.3227	2.3866	2.4386
10		1.9387	2.0361	2.1761	2.2901	2.4100	2.4822	2.5415
11		1.9868	2.0877	2.2343	2.3551	2.4844	2.5636	2.6310
12		2.0300	2.1340	2.2847	2.4115	2.5491	2.6353	2.7081
13		2.0689	2.1749	2.3306	2.4624	2.6077	2.6986	2.7779
14		2.1044	2.2125	2.3718	2.5069	2.6583	2.7557	2.8396
15		2.1367	2.2463	2.4087	2.5481	2.7048	2.8058	2.8947
16		2.1666	2.2780	2.4429	2.5853	2.7467	2.8525	2.9447
17		2.1943	2.3070	2.4749	2.6208	2.7853	2.8941	2.9906
18		2.2202	2.3339	2.5038	2.6518	2.8206	2.9323	3.0329
19		2.2443	2.3591	2.5312	2.6809	2.8533	2.9671	3.0708
20		2.2668	2.3825	2.5561	2.7081	2.8841	3.0008	3.1048
21		2.2879	2.4048	2.5799	2.7336	2.9119	3.0314	3.1397
22		2.3082	2.4257	2.6025	2.7582	2.9383	3.0596	3.1684
23		2.3272	2.4455	2.6235	2.7803	2.9639	3.0870	3.1983
24		2.3453	2.4642	2.6436	2.8013	2.9858	3.1114	3.2239
25		2.3622	2.4819	2.6625	2.8221	3.0090	3.1361	3.2508
26		2.3788	2.4987	2.6801	2.8411	3.0295	3.1577	3.2747
27		2.3946	2.5152	2.6977	2.8590	3.0498	3.1786	3.2975
28		2.4096	2.5303	2.7134	2.8760	3.0677	3.1983	3.3191
29		2.4238	2.5452	2.7290	2.8927	3.0851	3.2170	3.3381
30		2.4376	2.5593	2.7436	2.9081	3.1030	3.2361	3.3587
31		2.4508	2.5731	2.7583	2.9236	3.1195	3.2533	3.3770
32		2.4637	2.5864	2.7719	2.9375	3.1343	3.2693	3.3939
33		2.4758	2.5985	2.7853	2.9524	3.1495	3.2846	3.4098
34		2.4878	2.6110	2.7979	2.9650	3.1638	3.3005	3.4265
35		2.4993	2.6227	2.8101	2.9778	3.1783	3.3148	3.4431
36		2.5105	2.6339	2.8221	2.9902	3.1903	3.3284	3.4571
37		2.5211	2.6449	2.8332	3.0024	3.2036	3.3423	3.4710
38		2.5313	2.6556	2.8447	3.0139	3.2166	3.3548	3.4846
39		2.5418	2.6660	2.8551	3.0250	3.2272	3.3687	3.4993
40		2.5512	2.6757	2.8656	3.0361	3.2403	3.3811	3.5129
41		2.5610	2.6856	2.8752	3.0460	3.2500	3.3926	3.5244
42		2.5698	2.6948	2.8849	3.0559	3.2607	3.4020	3.5357
43		2.5791	2.7040	2.8945	3.0664	3.2722	3.4161	3.5485
44		2.5878	2.7131	2.9036	3.0756	3.2819	3.4250	3.5599
45		2.5962	2.7218	2.9130	3.0849	3.2912	3.4350	3.5702
46		2.6045	2.7305	2.9216	3.0938	3.3010	3.4454	3.5799
47		2.6125	2.7382	2.9300	3.1030	3.3105	3.4553	3.5914
48		2.6207	2.7463	2.9382	3.1117	3.3194	3.4639	3.5995
49		2.6283	2.7542	2.9464	3.1194	3.3277	3.4731	3.6108
50		2.6354	2.7613	2.9540	3.1276	3.3361	3.4808	3.6182
51		2.6431	2.7693	2.9617	3.1354	3.3457	3.4917	3.6294
52		2.6502	2.7764	2.9691	3.1433	3.3528	3.5002	3.6366
53		2.6574	2.7839	2.9765	3.1504	3.3610	3.5082	3.6467

continues

Table A2 (continued). Critical values for discordancy test N2 of an extreme outlier in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
54		2.6641	2.7906	2.9834	3.1581	3.3674	3.5157	3.6545
55		2.6706	2.7971	2.9903	3.1650	3.3752	3.5228	3.6601
56		2.6775	2.8039	2.9970	3.1721	3.3826	3.5308	3.6684
57		2.6838	2.8103	3.0039	3.1783	3.3894	3.5367	3.6769
58		2.6902	2.8169	3.0105	3.1855	3.3971	3.5448	3.6849
59		2.6964	2.8227	3.0167	3.1910	3.4037	3.5520	3.6924
60		2.7025	2.8294	3.0228	3.1986	3.4096	3.5590	3.7010
61		2.7084	2.8351	3.0288	3.2044	3.4171	3.5666	3.7058
62		2.7144	2.8412	3.0352	3.2111	3.4234	3.5728	3.7129
63		2.7199	2.8468	3.0410	3.2162	3.4297	3.5786	3.7196
64		2.7258	2.8527	3.0472	3.2220	3.4355	3.5852	3.7266
65		2.7314	2.8584	3.0526	3.2285	3.4419	3.5907	3.7329
66		2.7366	2.8636	3.0584	3.2343	3.4476	3.5973	3.7381
67		2.7420	2.8691	3.0641	3.2401	3.4540	3.6031	3.7450
68		2.7473	2.8744	3.0691	3.2450	3.4580	3.6091	3.7523
69		2.7524	2.8798	3.0739	3.2504	3.4646	3.6152	3.7565
70		2.7576	2.8848	3.0794	3.2559	3.4699	3.6209	3.7628
71		2.7625	2.8899	3.0842	3.2607	3.4757	3.6259	3.7684
72		2.7675	2.8946	3.0894	3.2657	3.4811	3.6317	3.7734
73		2.7722	2.8996	3.0947	3.2710	3.4856	3.6363	3.7795
74		2.7770	2.9043	3.0989	3.2756	3.4907	3.6419	3.7858
75		2.7818	2.9091	3.1038	3.2807	3.4957	3.6461	3.7887
76		2.7861	2.9137	3.1087	3.2855	3.5002	3.6517	3.7948
77		2.7910	2.9181	3.1130	3.2899	3.5052	3.6570	3.8017
78		2.7955	2.9226	3.1176	3.2946	3.5103	3.6613	3.8055
79		2.7997	2.9272	3.1225	3.2995	3.5152	3.6668	3.8100
80		2.8043	2.9318	3.1269	3.3039	3.5201	3.6708	3.8156
81		2.8084	2.9360	3.1313	3.3084	3.5244	3.6768	3.8217
82		2.8128	2.9402	3.1357	3.3126	3.5284	3.6807	3.8263
83		2.8171	2.9443	3.1397	3.3171	3.5330	3.6849	3.8291
84		2.8209	2.9483	3.1436	3.3208	3.5371	3.6897	3.8347
85		2.8252	2.9526	3.1478	3.3255	3.5420	3.6944	3.8404
86		2.8294	2.9566	3.1520	3.3293	3.5456	3.6977	3.8442
87		2.8332	2.9608	3.1560	3.3333	3.5505	3.7031	3.8479
88		2.8373	2.9647	3.1598	3.3374	3.5548	3.7072	3.8524
89		2.8411	2.9683	3.1639	3.3414	3.5580	3.7108	3.8573
90		2.8447	2.9722	3.1676	3.3454	3.5621	3.7145	3.8602
91		2.8484	2.9758	3.1715	3.3496	3.5658	3.7187	3.8654
92		2.8522	2.9797	3.1753	3.3532	3.5694	3.7223	3.8694
93		2.8559	2.9835	3.1794	3.3567	3.5737	3.7265	3.8736
94		2.8595	2.9871	3.1829	3.3607	3.5775	3.7306	3.8774
95		2.8631	2.9904	3.1863	3.3640	3.5811	3.7348	3.8805
96		2.8666	2.9940	3.1896	3.3675	3.5846	3.7386	3.8842
97		2.8702	2.9975	3.1931	3.3710	3.5881	3.7422	3.8884
98		2.8736	3.0011	3.1965	3.3743	3.5917	3.7457	3.8915
99		2.8770	3.0044	3.1997	3.3777	3.5959	3.7496	3.8960
100		2.8804	3.0079	3.2034	3.3816	3.5998	3.7526	3.8992

CL: Confidence level (%); SL: Significance level (%); α: Significance level. The headers for CL or SL or α generally used for most applications are given in italic-bold face. The mean values of the standard error of the mean ($\bar{x}_{se}$) for these critical values ($\bar{x}$) are (respective % errors are also reported in parentheses): ~0.00021 (for α=0.30, 0.008%); ~0.00024 (for α=0.20, 0.009%); ~0.00032 (for α=0.10, 0.011%); ~0.00041 (for α=0.05, 0.014%); ~0.0006 (for α=0.02, 0.017%); ~0.0007 (for α=0.01, 0.022%); and ~0.0010 (for α=0.005, 0.030%).

Table A3. Critical values for discordancy test N3 of an upper or lower pair (k=2; two outliers) in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
3		---	---	---	---	---	---	---
4		---	---	---	---	---	---	---
5		1.8893	1.9615	2.0470	2.1004	2.1419	2.1602	2.1716
6		2.1480	2.2295	2.3345	2.4078	2.4723	2.5046	2.5274
7		2.3599	2.4482	2.5677	2.6567	2.7412	2.7871	2.8214
8		2.5373	2.6323	2.7628	2.8644	2.9655	3.0231	3.0688
9		2.6874	2.7902	2.9303	3.0423	3.1575	3.2255	3.2813
10		2.8160	2.9265	3.0762	3.1967	3.3244	3.4017	3.4654
11		2.9279	3.0461	3.2049	3.3331	3.4707	3.5559	3.6283
12		3.0267	3.1517	3.3197	3.4548	3.6017	3.6943	3.7737
13		3.1153	3.2460	3.4225	3.5642	3.7189	3.8180	3.9035
14		3.1954	3.3315	3.5156	3.6636	3.8258	3.9304	4.0211
15		3.2686	3.4091	3.5999	3.7536	3.9228	4.0325	4.1294
16		3.3356	3.4800	3.6773	3.8364	4.0116	4.1254	4.2272
17		3.3978	3.5455	3.7484	3.9129	4.0942	4.2126	4.3180
18		3.4554	3.6064	3.8144	3.9834	4.1703	4.2920	4.4016
19		3.5096	3.6631	3.8756	4.0491	4.2411	4.3674	4.4798
20		3.5599	3.7161	3.9330	4.1102	4.3072	4.4365	4.5529
21		3.6074	3.7660	3.9863	4.1673	4.3685	4.5010	4.6208
22		3.6520	3.8122	4.0367	4.2212	4.4276	4.5626	4.6853
23		3.6945	3.8568	4.0841	4.2719	4.4815	4.6194	4.7450
24		3.7346	3.8989	4.1290	4.3199	4.5332	4.6744	4.8027
25		3.7732	3.9389	4.1718	4.3649	4.5824	4.7265	4.8571
26		3.8096	3.9767	4.2122	4.4078	4.6280	4.7739	4.9076
27		3.8446	4.0132	4.2509	4.4487	4.6722	4.8200	4.9549
28		3.8781	4.0477	4.2879	4.4877	4.7137	4.8644	5.0010
29		3.9099	4.0810	4.3229	4.5249	4.7539	4.9057	5.0447
30		3.9405	4.1125	4.3560	4.5604	4.7917	4.9462	5.0866
31		3.9701	4.1433	4.3885	4.5946	4.8280	4.9830	5.1265
32		3.9984	4.1724	4.4195	4.6274	4.8631	5.0202	5.1642
33		4.0257	4.2008	4.4498	4.6591	4.8966	5.0552	5.2011
34		4.0522	4.2277	4.4783	4.6891	4.9289	5.0899	5.2363
35		4.0774	4.2541	4.5060	4.7182	4.9599	5.1212	5.2703
36		4.1022	4.2794	4.5326	4.7457	4.9899	5.1538	5.3036
37		4.1261	4.3040	4.5583	4.7733	5.0181	5.1834	5.3346
38		4.1491	4.3278	4.5835	4.7992	5.0460	5.2125	5.3637
39		4.1717	4.3510	4.6077	4.8245	5.0729	5.2412	5.3947
40		4.1931	4.3734	4.6309	4.8491	5.0993	5.2680	5.4226
41		4.2143	4.3949	4.6530	4.8720	5.1243	5.2949	5.4507
42		4.2348	4.4159	4.6754	4.8955	5.1484	5.3188	5.4758
43		4.2547	4.4362	4.6966	4.9180	5.1721	5.3432	5.5027
44		4.2740	4.4559	4.7176	4.9396	5.1952	5.3682	5.5268
45		4.2931	4.4757	4.7376	4.9605	5.2174	5.3910	5.5502
46		4.3117	4.4947	4.7573	4.9810	5.2393	5.4127	5.5743
47		4.3293	4.5127	4.7763	5.0006	5.2600	5.4358	5.5981
48		4.3473	4.5308	4.7950	5.0205	5.2802	5.4564	5.6191
49		4.3641	4.5482	4.8135	5.0393	5.3000	5.4767	5.6401
50		4.3806	4.5652	4.8305	5.0574	5.3188	5.4955	5.6590
51		4.3973	4.5820	4.8481	5.0752	5.3374	5.5154	5.6808
52		4.4131	4.5984	4.8655	5.0933	5.3560	5.5358	5.7015
53		4.4285	4.6141	4.8812	5.1097	5.3742	5.5544	5.7206

continues

Table A3 (continued). Critical values for discordancy test N3 of an upper or lower pair (k=2; two outliers) in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
54		4.4439	4.6298	4.8976	5.1268	5.3923	5.5731	5.7402
55		4.4590	4.6449	4.9131	5.1424	5.4090	5.5896	5.7584
56		4.4736	4.6597	4.9287	5.1593	5.4253	5.6070	5.7749
57		4.4879	4.6744	4.9439	5.1743	5.4411	5.6239	5.7932
58		4.5021	4.6890	4.9587	5.1897	5.4562	5.6386	5.8083
59		4.5160	4.7028	4.9728	5.2044	5.4734	5.6567	5.8261
60		4.5293	4.7167	4.9874	5.2193	5.4886	5.6731	5.8428
61		4.5423	4.7299	5.0012	5.2341	5.5035	5.6879	5.8588
62		4.5554	4.7433	5.0151	5.2479	5.5184	5.7032	5.8747
63		4.5684	4.7564	5.0280	5.2613	5.5318	5.7173	5.8893
64		4.5811	4.7690	5.0410	5.2751	5.5465	5.7319	5.9036
65		4.5931	4.7812	5.0539	5.2886	5.5600	5.7456	5.9196
66		4.6051	4.7937	5.0667	5.3013	5.5741	5.7612	5.9352
67		4.6171	4.8055	5.0789	5.3140	5.5865	5.7734	5.9475
68		4.6287	4.8174	5.0915	5.3262	5.5994	5.7871	5.9605
69		4.6401	4.8288	5.1028	5.3387	5.6136	5.8007	5.9746
70		4.6518	4.8407	5.1144	5.3505	5.6252	5.8125	5.9869
71		4.6627	4.8519	5.1262	5.3622	5.6381	5.8258	6.0007
72		4.6738	4.8629	5.1373	5.3740	5.6500	5.8390	6.0148
73		4.6843	4.8740	5.1490	5.3853	5.6614	5.8502	6.0262
74		4.6950	4.8846	5.1597	5.3970	5.6725	5.8620	6.0382
75		4.7051	4.8951	5.1707	5.4078	5.6840	5.8739	6.0505
76		4.7155	4.9054	5.1810	5.4187	5.6956	5.8867	6.0616
77		4.7258	4.9159	5.1917	5.4295	5.7064	5.8961	6.0727
78		4.7357	4.9258	5.2019	5.4404	5.7172	5.9084	6.0857
79		4.7455	4.9358	5.2125	5.4508	5.7289	5.9192	6.0986
80		4.7553	4.9456	5.2223	5.4609	5.7394	5.9300	6.1088
81		4.7649	4.9557	5.2323	5.4713	5.7490	5.9408	6.1194
82		4.7743	4.9647	5.2417	5.4812	5.7599	5.9523	6.1311
83		4.7836	4.9742	5.2512	5.4901	5.7694	5.9616	6.1418
84		4.7926	4.9833	5.2604	5.5000	5.7798	5.9722	6.1521
85		4.8017	4.9926	5.2704	5.5101	5.7898	5.9813	6.1608
86		4.8106	5.0017	5.2789	5.5189	5.7995	5.9917	6.1714
87		4.8194	5.0104	5.2882	5.5284	5.8095	6.0026	6.1823
88		4.8282	5.0193	5.2972	5.5375	5.8179	6.0115	6.1917
89		4.8368	5.0277	5.3063	5.5464	5.8275	6.0206	6.2015
90		4.8451	5.0364	5.3145	5.5552	5.8369	6.0304	6.2104
91		4.8536	5.0447	5.3231	5.5642	5.8456	6.0400	6.2212
92		4.8617	5.0534	5.3319	5.5723	5.8544	6.0487	6.2293
93		4.8698	5.0614	5.3404	5.5813	5.8641	6.0577	6.2392
94		4.8779	5.0696	5.3482	5.5897	5.8717	6.0672	6.2477
95		4.8857	5.0775	5.3567	5.5979	5.8808	6.0746	6.2567
96		4.8936	5.0852	5.3647	5.6057	5.8882	6.0837	6.2662
97		4.9016	5.0934	5.3725	5.6140	5.8968	6.0914	6.2745
98		4.9091	5.1010	5.3801	5.6220	5.9052	6.0992	6.2816
99		4.9167	5.1084	5.3880	5.6299	5.9133	6.1089	6.2912
100		4.9242	5.1162	5.3958	5.6378	5.9210	6.1166	6.3003

CL: Confidence level (%); SL: Significance level (%); α: Significance level. The headers for CL or SL or α generally used for most applications are given in italic-bold face. The mean values of the standard error of the mean ($\bar{x}_{se}$) for these critical values ($\bar{x}$) are (respective % errors are also reported in parentheses): ~0.00021 (for α=0.30, 0.005%); ~0.00023 (for α=0.20, 0.005%); ~0.00028 (for α=0.10, 0.006%); ~0.00036 (for α=0.05, 0.007%); ~0.0005 (for α=0.02, 0.010%); ~0.0007 (for α=0.01, 0.012%); and ~0.0009 (for α=0.005, 0.017%).

Table A4. Critical values for discordancy test N3 of three upper or lower outliers (k=3) in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
3		---	---	---	---	---	---	---
4		---	---	---	---	---	---	---
5		---	---	---	---	---	---	---
6		---	---	---	---	---	---	---
7		2.7042	2.7818	2.8871	2.9658	3.0409	3.0813	3.1121
8		3.0067	3.0889	3.2025	3.2914	3.3802	3.4309	3.4711
9		3.2628	3.3524	3.4738	3.5710	3.6721	3.7322	3.7810
10		3.4823	3.5806	3.7120	3.8173	3.9294	3.9974	4.0545
11		3.6736	3.7808	3.9225	4.0355	4.1571	4.2327	4.2968
12		3.8425	3.9582	4.1104	4.2312	4.3617	4.4438	4.5153
13		3.9931	4.1164	4.2794	4.4076	4.5469	4.6357	4.7124
14		4.1291	4.2595	4.4325	4.5685	4.7156	4.8098	4.8922
15		4.2528	4.3894	4.5715	4.7153	4.8705	4.9705	5.0580
16		4.3658	4.5081	4.6989	4.8497	5.0129	5.1183	5.2110
17		4.4702	4.6182	4.8162	4.9737	5.1445	5.2538	5.3516
18		4.5667	4.7193	4.9250	5.0887	5.2665	5.3809	5.4829
19		4.6568	4.8140	5.0265	5.1964	5.3811	5.4999	5.6058
20		4.7405	4.9018	5.1204	5.2954	5.4864	5.6099	5.7196
21		4.8195	4.9845	5.2089	5.3890	5.5858	5.7140	5.8271
22		4.8934	5.0617	5.2914	5.4768	5.6792	5.8106	5.9291
23		4.9633	5.1350	5.3699	5.5600	5.7678	5.9032	6.0237
24		5.0298	5.2046	5.4441	5.6380	5.8520	5.9908	6.1159
25		5.0929	5.2703	5.5141	5.7121	5.9303	6.0725	6.2002
26		5.1526	5.3326	5.5802	5.7820	6.0049	6.1506	6.2810
27		5.2097	5.3923	5.6441	5.8495	6.0762	6.2250	6.3582
28		5.2645	5.4496	5.7044	5.9130	6.1445	6.2962	6.4336
29		5.3168	5.5035	5.7623	5.9739	6.2093	6.3634	6.5023
30		5.3668	5.5555	5.8169	6.0320	6.2704	6.4283	6.5703
31		5.4149	5.6053	5.8702	6.0869	6.3290	6.4879	6.6329
32		5.4610	5.6533	5.9208	6.1409	6.3871	6.5482	6.6945
33		5.5054	5.6995	5.9699	6.1924	6.4403	6.6044	6.7532
34		5.5484	5.7439	6.0168	6.2419	6.4938	6.6598	6.8101
35		5.5895	5.7863	6.0617	6.2891	6.5437	6.7116	6.8649
36		5.6294	5.8277	6.1055	6.3348	6.5926	6.7629	6.9171
37		5.6681	5.8678	6.1476	6.3793	6.6395	6.8111	6.9682
38		5.7056	5.9066	6.1883	6.4220	6.6847	6.8581	7.0158
39		5.7419	5.9443	6.2276	6.4631	6.7284	6.9038	7.0643
40		5.7769	5.9801	6.2657	6.5031	6.7712	6.9481	7.1104
41		5.8108	6.0151	6.3021	6.5413	6.8114	6.9911	7.1550
42		5.8444	6.0496	6.3380	6.5785	6.8503	7.0321	7.1959
43		5.8761	6.0826	6.3733	6.6158	6.8889	7.0701	7.2374
44		5.9076	6.1149	6.4070	6.6506	6.9262	7.1099	7.2772
45		5.9381	6.1463	6.4396	6.6850	6.9620	7.1480	7.3172
46		5.9676	6.1766	6.4715	6.7185	6.9979	7.1836	7.3537
47		5.9966	6.2060	6.5023	6.7505	7.0310	7.2191	7.3906
48		6.0249	6.2356	6.5328	6.7820	7.0656	7.2539	7.4274
49		6.0523	6.2635	6.5626	6.8130	7.0965	7.2868	7.4617
50		6.0789	6.2911	6.5911	6.8424	7.1281	7.3193	7.4942
51		6.1055	6.3181	6.6189	6.8714	7.1589	7.3505	7.5273
52		6.1308	6.3447	6.6467	6.9000	7.1884	7.3813	7.5584
53		6.1556	6.3698	6.6730	6.9279	7.2178	7.4121	7.5917

continues

Table A4 (continued). Critical values for discordancy test N3 of three upper or lower outliers (k=3) in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
54		6.1803	6.3950	6.6996	6.9553	7.2477	7.4433	7.6238
55		6.2044	6.4196	6.7246	6.9817	7.2743	7.4708	7.6512
56		6.2280	6.4439	6.7499	7.0079	7.3016	7.4990	7.6792
57		6.2508	6.4675	6.7747	7.0330	7.3279	7.5269	7.7104
58		6.2735	6.4905	6.7984	7.0574	7.3531	7.5516	7.7340
59		6.2953	6.5127	6.8216	7.0818	7.3784	7.5788	7.7630
60		6.3169	6.5351	6.8447	7.1057	7.4037	7.6046	7.7904
61		6.3378	6.5564	6.8673	7.1296	7.4288	7.6309	7.8158
62		6.3589	6.5779	6.8895	7.1517	7.4524	7.6543	7.8402
63		6.3793	6.5991	6.9106	7.1739	7.4748	7.6785	7.8657
64		6.3997	6.6194	6.9316	7.1955	7.4969	7.7009	7.8901
65		6.4192	6.6390	6.9530	7.2174	7.5208	7.7243	7.9134
66		6.4385	6.6591	6.9729	7.2388	7.5429	7.7482	7.9370
67		6.4570	6.6780	6.9928	7.2588	7.5643	7.7696	7.9585
68		6.4757	6.6973	7.0129	7.2794	7.5844	7.7905	7.9812
69		6.4939	6.7156	7.0322	7.2996	7.6062	7.8136	8.0052
70		6.5124	6.7341	7.0504	7.3182	7.6255	7.8336	8.0243
71		6.5299	6.7524	7.0695	7.3381	7.6466	7.8551	8.0479
72		6.5474	6.7702	7.0876	7.3565	7.6662	7.8748	8.0684
73		6.5647	6.7878	7.1054	7.3748	7.6849	7.8937	8.0884
74		6.5815	6.8046	7.1234	7.3937	7.7030	7.9149	8.1085
75		6.5979	6.8216	7.1409	7.4114	7.7212	7.9328	8.1274
76		6.6141	6.8380	7.1575	7.4292	7.7410	7.9526	8.1473
77		6.6304	6.8547	7.1752	7.4462	7.7586	7.9694	8.1639
78		6.6464	6.8711	7.1916	7.4632	7.7766	7.9884	8.1839
79		6.6620	6.8870	7.2082	7.4807	7.7944	8.0062	8.2026
80		6.6775	6.9023	7.2243	7.4973	7.8114	8.0252	8.2211
81		6.6928	6.9182	7.2405	7.5140	7.8284	8.0418	8.2388
82		6.7076	6.9331	7.2557	7.5295	7.8450	8.0598	8.2572
83		6.7221	6.9481	7.2708	7.5452	7.8610	8.0760	8.2750
84		6.7368	6.9627	7.2861	7.5612	7.8780	8.0936	8.2921
85		6.7513	6.9776	7.3015	7.5767	7.8925	8.1068	8.3063
86		6.7653	6.9918	7.3157	7.5915	7.9094	8.1256	8.3253
87		6.7793	7.0060	7.3306	7.6064	7.9248	8.1414	8.3424
88		6.7933	7.0200	7.3448	7.6215	7.9400	8.1568	8.3570
89		6.8070	7.0339	7.3592	7.6355	7.9538	8.1705	8.3724
90		6.8203	7.0475	7.3732	7.6499	7.9694	8.1870	8.3906
91		6.8336	7.0610	7.3869	7.6639	7.9842	8.2016	8.4040
92		6.8465	7.0740	7.4007	7.6781	7.9982	8.2164	8.4189
93		6.8595	7.0873	7.4137	7.6916	8.0131	8.2320	8.4353
94		6.8723	7.1006	7.4272	7.7052	8.0263	8.2450	8.4480
95		6.8849	7.1133	7.4402	7.7187	8.0409	8.2585	8.4627
96		6.8971	7.1255	7.4531	7.7318	8.0540	8.2739	8.4782
97		6.9097	7.1381	7.4658	7.7445	8.0669	8.2871	8.4922
98		6.9218	7.1504	7.4783	7.7578	8.0801	8.2997	8.5033
99		6.9339	7.1627	7.4909	7.7707	8.0938	8.3144	8.5185
100		6.9457	7.1744	7.5033	7.7827	8.1073	8.3278	8.5320

CL: Confidence level (%); SL: Significance level (%); α: Significance level. The headers for CL or SL or α generally used for most applications are given in italic-bold face. The mean values of the standard error of the mean ($\bar{x}_{se}$) for these critical values ($\bar{x}$) are (respective % errors are also reported in parentheses): ~0.00024 (for α=0.30, 0.0041%); ~0.00026 (for α=0.20, 0.0043%); ~0.00032 (for α=0.10, 0.005%); ~0.00041 (for α=0.05, 0.006%); ~0.0005 (for α=0.02, 0.008%); ~0.0007 (for α=0.01, 0.010%); and ~0.0010 (for α=0.005, 0.014%).

Table A5. Critical values for discordancy test N3 of four upper or lower outliers (k=4) in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
3		---	---	---	---	---	---	---
4		---	---	---	---	---	---	---
5		---	---	---	---	---	---	---
6		---	---	---	---	---	---	---
7		---	---	---	---	---	---	---
8		---	---	---	---	---	---	---
9		3.5289	3.6116	3.7246	3.8155	3.9097	3.9654	4.0114
10		3.8507	3.9404	4.0606	4.1572	4.2601	4.3230	4.3756
11		4.1311	4.2286	4.3577	4.4606	4.5717	4.6411	4.6996
12		4.3784	4.4840	4.6231	4.7333	4.8518	4.9273	4.9927
13		4.5990	4.7126	4.8622	4.9794	5.1063	5.1872	5.2577
14		4.7979	4.9190	5.0787	5.2038	5.3385	5.4249	5.5010
15		4.9783	5.1067	5.2762	5.4092	5.5521	5.6436	5.7244
16		5.1431	5.2785	5.4577	5.5984	5.7491	5.8460	5.9317
17		5.2947	5.4364	5.6247	5.7727	5.9318	6.0334	6.1241
18		5.4347	5.5824	5.7790	5.9338	6.1006	6.2073	6.3023
19		5.5652	5.7185	5.9235	6.0856	6.2604	6.3721	6.4712
20		5.6866	5.8453	6.0574	6.2258	6.4071	6.5242	6.6282
21		5.8003	5.9638	6.1834	6.3577	6.5461	6.6680	6.7754
22		5.9070	6.0753	6.3018	6.4818	6.6768	6.8020	6.9146
23		6.0076	6.1802	6.4132	6.5988	6.8012	6.9310	7.0462
24		6.1029	6.2797	6.5185	6.7096	6.9176	7.0520	7.1718
25		6.1934	6.3739	6.6183	6.8143	7.0280	7.1662	7.2899
26		6.2790	6.4627	6.7126	6.9136	7.1325	7.2750	7.4018
27		6.3608	6.5482	6.8032	7.0078	7.2329	7.3792	7.5092
28		6.4389	6.6296	6.8891	7.0984	7.3289	7.4785	7.6126
29		6.5136	6.7069	6.9710	7.1846	7.4196	7.5729	7.7094
30		6.5848	6.7808	7.0498	7.2670	7.5063	7.6621	7.8012
31		6.6534	6.8521	7.1244	7.3451	7.5881	7.7472	7.8902
32		6.7190	6.9204	7.1966	7.4216	7.6692	7.8311	7.9752
33		6.7822	6.9859	7.2660	7.4940	7.7457	7.9106	8.0586
34		6.8430	7.0491	7.3328	7.5641	7.8196	7.9867	8.1378
35		6.9016	7.1095	7.3963	7.6305	7.8904	8.0607	8.2133
36		6.9583	7.1683	7.4582	7.6958	7.9588	8.1309	8.2868
37		7.0131	7.2251	7.5183	7.7576	8.0234	8.1992	8.3579
38		7.0663	7.2801	7.5759	7.8185	8.0881	8.2657	8.4256
39		7.1176	7.3334	7.6314	7.8765	8.1499	8.3295	8.4914
40		7.1668	7.3843	7.6858	7.9328	8.2088	8.3915	8.5568
41		7.2151	7.4340	7.7378	7.9876	8.2665	8.4504	8.6157
42		7.2622	7.4826	7.7886	8.0401	8.3217	8.5075	8.6761
43		7.3073	7.5290	7.8380	8.0917	8.3760	8.5643	8.7338
44		7.3517	7.5750	7.8854	8.1415	8.4282	8.6183	8.7905
45		7.3949	7.6194	7.9319	8.1901	8.4795	8.6715	8.8463
46		7.4366	7.6622	7.9768	8.2375	8.5288	8.7223	8.8994
47		7.4770	7.7041	8.0207	8.2822	8.5766	8.7717	8.9485
48		7.5171	7.7453	8.0640	8.3277	8.6240	8.8204	8.9987
49		7.5557	7.7852	8.1055	8.3710	8.6686	8.8667	9.0463
50		7.5934	7.8239	8.1461	8.4132	8.7132	8.9133	9.0936
51		7.6305	7.8620	8.1853	8.4540	8.7570	8.9575	9.1410
52		7.6662	7.8990	8.2242	8.4945	8.7982	9.0000	9.1846
53		7.7014	7.9349	8.2615	8.5327	8.8397	9.0434	9.2293

continues

Table A5 (continued). Critical values for discordancy test N3 of four upper or lower outliers (k=4) in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
54		7.7357	7.9704	8.2987	8.5720	8.8801	9.0846	9.2711
55		7.7694	8.0048	8.3344	8.6091	8.9191	9.1246	9.3137
56		7.8027	8.0392	8.3701	8.6456	8.9581	9.1644	9.3548
57		7.8350	8.0721	8.4046	8.6815	8.9956	9.2037	9.3950
58		7.8666	8.1046	8.4383	8.7166	9.0308	9.2410	9.4317
59		7.8976	8.1363	8.4710	8.7504	9.0667	9.2776	9.4705
60		7.9277	8.1671	8.5035	8.7842	9.1012	9.3136	9.5074
61		7.9572	8.1976	8.5352	8.8174	9.1365	9.3498	9.5447
62		7.9867	8.2275	8.5660	8.8493	9.1699	9.3840	9.5788
63		8.0156	8.2569	8.5969	8.8805	9.2018	9.4176	9.6147
64		8.0436	8.2856	8.6265	8.9109	9.2337	9.4501	9.6491
65		8.0709	8.3138	8.6550	8.9412	9.2663	9.4838	9.6837
66		8.0981	8.3416	8.6841	8.9715	9.2969	9.5154	9.7146
67		8.1242	8.3681	8.7124	9.0007	9.3275	9.5469	9.7467
68		8.1503	8.3950	8.7401	9.0292	9.3561	9.5754	9.7795
69		8.1757	8.4212	8.7675	9.0579	9.3866	9.6084	9.8095
70		8.2016	8.4470	8.7936	9.0837	9.4141	9.6358	9.8394
71		8.2261	8.4724	8.8203	9.1118	9.4428	9.6660	9.8701
72		8.2504	8.4976	8.8458	9.1378	9.4701	9.6947	9.8996
73		8.2745	8.5220	8.8713	9.1640	9.4974	9.7224	9.9274
74		8.2983	8.5460	8.8958	9.1895	9.5249	9.7493	9.9571
75		8.3212	8.5697	8.9206	9.2148	9.5502	9.7755	9.9833
76		8.3440	8.5926	8.9446	9.2401	9.5770	9.8028	10.0120
77		8.3666	8.6159	8.9685	9.2646	9.6017	9.8288	10.0370
78		8.3889	8.6385	8.9916	9.2885	9.6260	9.8544	10.0631
79		8.4106	8.6610	9.0150	9.3126	9.6509	9.8802	10.0891
80		8.4323	8.6828	9.0381	9.3362	9.6763	9.9047	10.1150
81		8.4535	8.7049	9.0604	9.3586	9.6999	9.9286	10.1400
82		8.4745	8.7256	9.0815	9.3812	9.7230	9.9535	10.1670
83		8.4946	8.7467	9.1032	9.4030	9.7462	9.9776	10.1890
84		8.5149	8.7670	9.1244	9.4258	9.7702	10.0010	10.2150
85		8.5353	8.7877	9.1460	9.4468	9.7899	10.0210	10.2341
86		8.5547	8.8080	9.1667	9.4684	9.8133	10.0470	10.2620
87		8.5743	8.8275	9.1869	9.4896	9.8355	10.0681	10.2850
88		8.5936	8.8474	9.2071	9.5100	9.8568	10.0920	10.3071
89		8.6129	8.8670	9.2271	9.5304	9.8768	10.1110	10.3261
90		8.6315	8.8859	9.2461	9.5505	9.8984	10.1340	10.3511
91		8.6500	8.9046	9.2659	9.5701	9.9187	10.1531	10.3710
92		8.6680	8.9228	9.2850	9.5906	9.9396	10.1751	10.3940
93		8.6860	8.9415	9.3036	9.6095	9.9596	10.1961	10.4141
94		8.7042	8.9598	9.3223	9.6285	9.9787	10.2150	10.4340
95		8.7217	8.9774	9.3406	9.6471	9.9979	10.2351	10.4561
96		8.7386	8.9946	9.3588	9.6658	10.0171	10.2541	10.4751
97		8.7563	9.0124	9.3767	9.6838	10.0360	10.2731	10.4951
98		8.7731	9.0297	9.3943	9.7019	10.0540	10.2921	10.5140
99		8.7896	9.0468	9.4119	9.7202	10.0740	10.3120	10.5331
100		8.8064	9.0631	9.4287	9.7374	10.0911	10.3301	10.5521

CL: Confidence level (%); SL: Significance level (%); α: Significance level. The headers for CL or SL or α generally used for most applications are given in italic-bold face. The mean values of the standard error of the mean ($\bar{x}_{se}$) for these critical values ($\bar{x}$) are (respective % errors are also reported in parentheses): ~0.00026 (for α=0.30, 0.0036%); ~0.00030 (for α=0.20, 0.0039%); ~0.00035 (for α=0.10, 0.0044%); ~0.00044 (for α=0.05, 0.005%); ~0.0006 (for α=0.02, 0.007%); ~0.0008 (for α=0.01, 0.009%); and ~0.0010 (for α=0.005, 0.011%).

Table A6. Critical values for discordancy test N4 of an upper or lower outlier (k=1) in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
3		0.0955	0.0432	0.0109	0.0027	0.0004	0.0001	0.0000
4		0.2776	0.1900	0.0976	0.0494	0.0199	0.0100	0.0050
5		0.3922	0.3065	0.1983	0.1270	0.0698	0.0442	0.0279
6		0.4680	0.3898	0.2825	0.2031	0.1303	0.0928	0.0659
7		0.5227	0.4520	0.3501	0.2694	0.1895	0.1448	0.1104
8		0.5646	0.5005	0.4049	0.3261	0.2434	0.1946	0.1555
9		0.5980	0.5394	0.4501	0.3740	0.2915	0.2411	0.1990
10		0.6255	0.5715	0.4881	0.4154	0.3343	0.2831	0.2394
11		0.6489	0.5987	0.5205	0.4510	0.3718	0.3209	0.2769
12		0.6689	0.6219	0.5483	0.4823	0.4058	0.3556	0.3113
13		0.6863	0.6422	0.5727	0.5096	0.4355	0.3861	0.3423
14		0.7017	0.6600	0.5942	0.5340	0.4626	0.4146	0.3711
15		0.7154	0.6759	0.6134	0.5559	0.4870	0.4401	0.3974
16		0.7276	0.6901	0.6305	0.5756	0.5091	0.4635	0.4215
17		0.7387	0.7030	0.6461	0.5932	0.5289	0.4847	0.4439
18		0.7487	0.7146	0.6602	0.6095	0.5474	0.5044	0.4646
19		0.7579	0.7252	0.6731	0.6242	0.5644	0.5226	0.4836
20		0.7663	0.7350	0.6849	0.6379	0.5800	0.5393	0.5013
21		0.7740	0.7439	0.6958	0.6504	0.5944	0.5549	0.5176
22		0.7812	0.7522	0.7058	0.6620	0.6077	0.5693	0.5330
23		0.7879	0.7599	0.7151	0.6728	0.6200	0.5826	0.5473
24		0.7941	0.7671	0.7238	0.6828	0.6318	0.5955	0.5607
25		0.7999	0.7738	0.7319	0.6921	0.6425	0.6070	0.5732
26		0.8053	0.7801	0.7395	0.7010	0.6527	0.6182	0.5853
27		0.8103	0.7859	0.7465	0.7092	0.6623	0.6285	0.5965
28		0.8151	0.7914	0.7533	0.7170	0.6714	0.6386	0.6072
29		0.8196	0.7967	0.7596	0.7243	0.6798	0.6478	0.6172
30		0.8239	0.8016	0.7656	0.7313	0.6879	0.6566	0.6265
31		0.8279	0.8062	0.7712	0.7377	0.6954	0.6648	0.6355
32		0.8318	0.8106	0.7765	0.7440	0.7028	0.6729	0.6442
33		0.8354	0.8148	0.7816	0.7497	0.7094	0.6803	0.6523
34		0.8389	0.8188	0.7864	0.7554	0.7161	0.6875	0.6600
35		0.8422	0.8226	0.7909	0.7607	0.7222	0.6942	0.6673
36		0.8453	0.8262	0.7953	0.7657	0.7282	0.7009	0.6744
37		0.8483	0.8296	0.7995	0.7706	0.7338	0.7070	0.6810
38		0.8512	0.8330	0.8034	0.7752	0.7391	0.7128	0.6876
39		0.8539	0.8361	0.8072	0.7796	0.7443	0.7187	0.6935
40		0.8566	0.8392	0.8109	0.7838	0.7493	0.7239	0.6994
41		0.8591	0.8420	0.8144	0.7879	0.7541	0.7294	0.7051
42		0.8615	0.8448	0.8178	0.7918	0.7586	0.7343	0.7108
43		0.8639	0.8475	0.8210	0.7955	0.7629	0.7390	0.7156
44		0.8661	0.8500	0.8240	0.7991	0.7672	0.7437	0.7208
45		0.8683	0.8525	0.8270	0.8025	0.7712	0.7482	0.7258
46		0.8703	0.8549	0.8299	0.8059	0.7751	0.7525	0.7303
47		0.8724	0.8572	0.8327	0.8090	0.7788	0.7566	0.7347
48		0.8743	0.8594	0.8353	0.8121	0.7824	0.7606	0.7393
49		0.8762	0.8616	0.8379	0.8151	0.7859	0.7644	0.7434
50		0.8780	0.8637	0.8404	0.8180	0.7893	0.7682	0.7476
51		0.8797	0.8656	0.8428	0.8208	0.7926	0.7716	0.7513
52		0.8814	0.8676	0.8451	0.8235	0.7957	0.7753	0.7551
53		0.8831	0.8695	0.8473	0.8261	0.7988	0.7786	0.7587

Continues

Table A6 (continued). Critical values for discordancy test N4 of an upper or lower outlier (k=1) in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
54		0.8847	0.8713	0.8495	0.8286	0.8018	0.7820	0.7624
55		0.8862	0.8730	0.8517	0.8311	0.8046	0.7851	0.7659
56		0.8877	0.8747	0.8537	0.8334	0.8074	0.7881	0.7692
57		0.8892	0.8764	0.8557	0.8357	0.8101	0.7912	0.7726
58		0.8906	0.8780	0.8576	0.8380	0.8127	0.7940	0.7756
59		0.8920	0.8796	0.8595	0.8401	0.8153	0.7968	0.7787
60		0.8933	0.8811	0.8613	0.8422	0.8177	0.7996	0.7817
61		0.8946	0.8826	0.8631	0.8443	0.8201	0.8022	0.7845
62		0.8958	0.8840	0.8648	0.8462	0.8224	0.8047	0.7873
63		0.8971	0.8854	0.8665	0.8482	0.8247	0.8072	0.7900
64		0.8982	0.8867	0.8680	0.8500	0.8269	0.8097	0.7928
65		0.8994	0.8881	0.8696	0.8519	0.8290	0.8120	0.7953
66		0.9006	0.8894	0.8712	0.8536	0.8310	0.8143	0.7979
67		0.9017	0.8906	0.8727	0.8553	0.8330	0.8165	0.8003
68		0.9027	0.8918	0.8741	0.8571	0.8351	0.8188	0.8027
69		0.9038	0.8930	0.8756	0.8587	0.8370	0.8209	0.8049
70		0.9048	0.8942	0.8769	0.8603	0.8388	0.8230	0.8073
71		0.9058	0.8953	0.8783	0.8619	0.8407	0.8249	0.8095
72		0.9068	0.8964	0.8796	0.8634	0.8425	0.8269	0.8116
73		0.9077	0.8975	0.8809	0.8649	0.8443	0.8289	0.8138
74		0.9087	0.8986	0.8822	0.8664	0.8459	0.8307	0.8158
75		0.9096	0.8996	0.8834	0.8678	0.8476	0.8326	0.8179
76		0.9105	0.9006	0.8846	0.8692	0.8492	0.8344	0.8198
77		0.9114	0.9016	0.8858	0.8706	0.8508	0.8362	0.8217
78		0.9122	0.9026	0.8870	0.8719	0.8524	0.8379	0.8236
79		0.9130	0.9035	0.8881	0.8732	0.8539	0.8396	0.8254
80		0.9139	0.9045	0.8892	0.8744	0.8554	0.8411	0.8272
81		0.9147	0.9054	0.8902	0.8757	0.8568	0.8428	0.8289
82		0.9154	0.9062	0.8913	0.8769	0.8582	0.8444	0.8307
83		0.9162	0.9071	0.8923	0.8781	0.8596	0.8459	0.8324
84		0.9170	0.9080	0.8934	0.8792	0.8610	0.8474	0.8340
85		0.9177	0.9088	0.8943	0.8804	0.8623	0.8488	0.8356
86		0.9184	0.9096	0.8953	0.8815	0.8636	0.8503	0.8372
87		0.9191	0.9104	0.8963	0.8826	0.8649	0.8517	0.8386
88		0.9198	0.9112	0.8972	0.8837	0.8661	0.8531	0.8402
89		0.9205	0.9120	0.8981	0.8847	0.8674	0.8545	0.8417
90		0.9212	0.9127	0.8990	0.8858	0.8686	0.8558	0.8432
91		0.9218	0.9135	0.8999	0.8868	0.8697	0.8571	0.8446
92		0.9225	0.9142	0.9007	0.8877	0.8709	0.8584	0.8461
93		0.9231	0.9149	0.9016	0.8887	0.8721	0.8596	0.8474
94		0.9237	0.9156	0.9024	0.8897	0.8732	0.8609	0.8487
95		0.9244	0.9163	0.9032	0.8906	0.8743	0.8621	0.8500
96		0.9250	0.9170	0.9041	0.8915	0.8754	0.8633	0.8513
97		0.9255	0.9176	0.9048	0.8924	0.8764	0.8645	0.8526
98		0.9261	0.9183	0.9056	0.8933	0.8774	0.8656	0.8539
99		0.9267	0.9189	0.9064	0.8942	0.8785	0.8667	0.8551
100		0.9272	0.9196	0.9071	0.8951	0.8794	0.8678	0.8563

CL: Confidence level (%); SL: Significance level (%); α: Significance level. The headers for CL or SL or α generally used for most applications are given in italic-bold face. The mean values of the standard error of the mean ($\bar{x}_{se}$) for these critical values ($\bar{x}$) are (respective % errors are also reported in parentheses): ~0.000018 (for α=0.30, 0.0032%); ~0.000021 (for α=0.20, 0.0042%); ~0.000028 (for α=0.10, 0.007%); ~0.000037 (for α=0.05, 0.010%); ~0.00005 (for α=0.02, 0.016%); ~0.00007 (for α=0.01, 0.023%); and ~0.00010 (for α=0.005, 0.027%).

Table A7. Critical values for discordancy test N4 of two upper or lower outliers (k=2) in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
3		---	---	---	---	---	---	---
4		0.0302	0.0129	0.0031	0.0008	0.0001	0.0000	0.0000
5		0.1225	0.0786	0.0377	0.0184	0.0072	0.0035	0.0017
6		0.2058	0.1518	0.0921	0.0564	0.0299	0.0184	0.0115
7		0.2730	0.2163	0.1477	0.1019	0.0628	0.0437	0.0303
8		0.3275	0.2713	0.1991	0.1476	0.1002	0.0750	0.0564
9		0.3731	0.3180	0.2452	0.1908	0.1382	0.1083	0.0853
10		0.4114	0.3583	0.2863	0.2306	0.1743	0.1418	0.1158
11		0.4443	0.3934	0.3229	0.2670	0.2090	0.1740	0.1452
12		0.4730	0.4241	0.3551	0.2998	0.2412	0.2045	0.1746
13		0.4982	0.4511	0.3843	0.3294	0.2704	0.2336	0.2017
14		0.5207	0.4755	0.4105	0.3564	0.2973	0.2601	0.2283
15		0.5407	0.4972	0.4345	0.3817	0.3236	0.2861	0.2533
16		0.5590	0.5169	0.4561	0.4047	0.3476	0.3104	0.2773
17		0.5755	0.5351	0.4761	0.4259	0.3692	0.3320	0.2992
18		0.5907	0.5516	0.4942	0.4452	0.3898	0.3533	0.3207
19		0.6043	0.5666	0.5112	0.4633	0.4088	0.3724	0.3396
20		0.6173	0.5807	0.5267	0.4804	0.4267	0.3909	0.3588
21		0.6291	0.5937	0.5412	0.4958	0.4434	0.4080	0.3763
22		0.6402	0.6059	0.5548	0.5105	0.4585	0.4239	0.3918
23		0.6504	0.6171	0.5675	0.5244	0.4739	0.4395	0.4080
24		0.6599	0.6276	0.5793	0.5371	0.4876	0.4542	0.4234
25		0.6688	0.6375	0.5907	0.5494	0.5007	0.4674	0.4367
26		0.6772	0.6467	0.6010	0.5608	0.5135	0.4809	0.4509
27		0.6851	0.6554	0.6108	0.5717	0.5253	0.4934	0.4637
28		0.6926	0.6636	0.6201	0.5818	0.5365	0.5053	0.4765
29		0.6997	0.6715	0.6290	0.5916	0.5471	0.5166	0.4874
30		0.7063	0.6789	0.6375	0.6010	0.5571	0.5268	0.4990
31		0.7127	0.6858	0.6453	0.6095	0.5667	0.5370	0.5093
32		0.7188	0.6925	0.6529	0.6176	0.5755	0.5468	0.5197
33		0.7245	0.6989	0.6600	0.6256	0.5845	0.5561	0.5294
34		0.7300	0.7049	0.6670	0.6331	0.5924	0.5640	0.5382
35		0.7352	0.7108	0.6735	0.6404	0.6007	0.5732	0.5470
36		0.7402	0.7162	0.6798	0.6474	0.6084	0.5812	0.5557
37		0.7449	0.7214	0.6858	0.6539	0.6155	0.5888	0.5633
38		0.7495	0.7266	0.6917	0.6605	0.6226	0.5963	0.5717
39		0.7539	0.7314	0.6971	0.6664	0.6295	0.6036	0.5794
40		0.7582	0.7360	0.7024	0.6722	0.6357	0.6102	0.5861
41		0.7622	0.7405	0.7075	0.6780	0.6423	0.6171	0.5929
42		0.7661	0.7448	0.7124	0.6834	0.6482	0.6234	0.6002
43		0.7698	0.7489	0.7172	0.6887	0.6541	0.6294	0.6064
44		0.7734	0.7530	0.7216	0.6935	0.6594	0.6351	0.6125
45		0.7769	0.7568	0.7261	0.6984	0.6648	0.6413	0.6190
46		0.7802	0.7604	0.7302	0.7031	0.6702	0.6469	0.6245
47		0.7835	0.7641	0.7344	0.7077	0.6751	0.6520	0.6303
48		0.7866	0.7675	0.7384	0.7120	0.6800	0.6574	0.6360
49		0.7897	0.7709	0.7422	0.7163	0.6847	0.6622	0.6411
50		0.7926	0.7741	0.7458	0.7203	0.6893	0.6675	0.6466
51		0.7954	0.7772	0.7494	0.7243	0.6938	0.6721	0.6512
52		0.7982	0.7803	0.7529	0.7282	0.6981	0.6767	0.6563
53		0.8009	0.7832	0.7563	0.7319	0.7023	0.6811	0.6610

continues

Table A7 (continued). Critical values for discordancy test N4 of two upper or lower outliers (k=2) in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
54		0.8034	0.7861	0.7595	0.7355	0.7062	0.6854	0.6655
55		0.8060	0.7888	0.7627	0.7390	0.7102	0.6896	0.6699
56		0.8084	0.7915	0.7657	0.7423	0.7139	0.6938	0.6742
57		0.8107	0.7941	0.7687	0.7458	0.7178	0.6976	0.6783
58		0.8130	0.7967	0.7716	0.7489	0.7213	0.7017	0.6826
59		0.8153	0.7992	0.7745	0.7521	0.7245	0.7050	0.6863
60		0.8175	0.8015	0.7771	0.7551	0.7280	0.7089	0.6903
61		0.8196	0.8039	0.7798	0.7580	0.7313	0.7122	0.6942
62		0.8217	0.8062	0.7825	0.7609	0.7345	0.7157	0.6977
63		0.8237	0.8084	0.7850	0.7637	0.7376	0.7188	0.7012
64		0.8257	0.8105	0.7874	0.7664	0.7407	0.7225	0.7051
65		0.8276	0.8127	0.7897	0.7691	0.7438	0.7258	0.7084
66		0.8294	0.8147	0.7921	0.7716	0.7465	0.7286	0.7112
67		0.8312	0.8167	0.7944	0.7740	0.7494	0.7317	0.7144
68		0.8330	0.8186	0.7966	0.7767	0.7522	0.7347	0.7179
69		0.8348	0.8206	0.7989	0.7790	0.7546	0.7375	0.7211
70		0.8364	0.8225	0.8010	0.7814	0.7574	0.7405	0.7242
71		0.8381	0.8242	0.8030	0.7837	0.7601	0.7432	0.7271
72		0.8397	0.8260	0.8050	0.7859	0.7624	0.7455	0.7295
73		0.8413	0.8278	0.8070	0.7881	0.7649	0.7483	0.7325
74		0.8428	0.8294	0.8089	0.7902	0.7673	0.7508	0.7349
75		0.8443	0.8311	0.8108	0.7923	0.7698	0.7537	0.7379
76		0.8458	0.8327	0.8126	0.7944	0.7719	0.7558	0.7404
77		0.8472	0.8343	0.8145	0.7963	0.7743	0.7585	0.7432
78		0.8486	0.8359	0.8162	0.7983	0.7764	0.7607	0.7453
79		0.8500	0.8374	0.8179	0.8002	0.7785	0.7629	0.7477
80		0.8514	0.8389	0.8196	0.8022	0.7807	0.7653	0.7504
81		0.8527	0.8403	0.8212	0.8039	0.7826	0.7674	0.7528
82		0.8540	0.8418	0.8229	0.8058	0.7848	0.7696	0.7550
83		0.8553	0.8431	0.8246	0.8076	0.7868	0.7717	0.7570
84		0.8565	0.8445	0.8261	0.8093	0.7887	0.7736	0.7593
85		0.8578	0.8459	0.8276	0.8109	0.7905	0.7757	0.7616
86		0.8590	0.8472	0.8291	0.8127	0.7924	0.7780	0.7638
87		0.8601	0.8485	0.8306	0.8143	0.7943	0.7798	0.7658
88		0.8613	0.8497	0.8321	0.8158	0.7961	0.7817	0.7679
89		0.8624	0.8510	0.8334	0.8174	0.7979	0.7836	0.7697
90		0.8635	0.8522	0.8348	0.8190	0.7995	0.7854	0.7719
91		0.8646	0.8534	0.8362	0.8205	0.8012	0.7871	0.7736
92		0.8657	0.8546	0.8375	0.8219	0.8029	0.7890	0.7757
93		0.8668	0.8558	0.8388	0.8235	0.8044	0.7908	0.7775
94		0.8678	0.8569	0.8401	0.8248	0.8061	0.7925	0.7796
95		0.8688	0.8580	0.8414	0.8262	0.8076	0.7940	0.7809
96		0.8699	0.8592	0.8427	0.8277	0.8092	0.7958	0.7830
97		0.8708	0.8602	0.8439	0.8290	0.8107	0.7974	0.7846
98		0.8718	0.8613	0.8451	0.8304	0.8121	0.7991	0.7865
99		0.8727	0.8623	0.8463	0.8317	0.8136	0.8006	0.7880
100		0.8737	0.8633	0.8475	0.8330	0.8151	0.8022	0.7897

CL: Confidence level (%); SL: Significance level (%); α: Significance level. The headers for CL or SL or α generally used for most applications are given in italic-bold face. The mean values of the standard error of the mean ($\bar{x}_{se}$) for these critical values ($\bar{x}$) are (respective % errors are also reported in parentheses): ~0.00007 (for α=0.30, 0.016%); ~0.00008 (for α=0.20, 0.021%); ~0.00010 (for α=0.10, 0.029%); ~0.00012 (for α=0.05, 0.036%); ~0.00017 (for α=0.02, 0.06%); ~0.00022 (for α=0.01, 0.07%); and ~0.00027 (for α=0.005, 0.09%).

Table A8. Critical values for discordancy test N4 of three upper or lower outliers (k=3) in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
3		---	---	---	---	---	---	---
4		---	---	---	---	---	---	---
5		---	---	---	---	---	---	---
6		0.0713	0.0445	0.0207	0.0099	0.0038	0.0019	0.0009
7		0.1306	0.0937	0.0550	0.0330	0.0172	0.0106	0.0066
8		0.1841	0.1423	0.0941	0.0635	0.0386	0.0265	0.0184
9		0.2310	0.1869	0.1333	0.0968	0.0642	0.0476	0.0354
10		0.2721	0.2271	0.1703	0.1298	0.0919	0.0715	0.0557
11		0.3083	0.2632	0.2050	0.1620	0.1203	0.0964	0.0778
12		0.3403	0.2956	0.2366	0.1921	0.1475	0.1216	0.1009
13		0.3690	0.3249	0.2660	0.2207	0.1741	0.1466	0.1237
14		0.3949	0.3517	0.2932	0.2473	0.1995	0.1706	0.1463
15		0.4181	0.3759	0.3181	0.2722	0.2238	0.1942	0.1691
16		0.4394	0.3981	0.3413	0.2954	0.2467	0.2165	0.1902
17		0.4588	0.4185	0.3628	0.3175	0.2685	0.2374	0.2104
18		0.4766	0.4375	0.3826	0.3379	0.2889	0.2581	0.2315
19		0.4929	0.4549	0.4011	0.3569	0.3080	0.2765	0.2493
20		0.5084	0.4710	0.4184	0.3749	0.3265	0.2953	0.2677
21		0.5224	0.4862	0.4347	0.3918	0.3437	0.3126	0.2849
22		0.5359	0.5005	0.4499	0.4076	0.3598	0.3287	0.3009
23		0.5483	0.5137	0.4642	0.4227	0.3757	0.3452	0.3172
24		0.5599	0.5261	0.4776	0.4369	0.3905	0.3603	0.3328
25		0.5707	0.5379	0.4904	0.4502	0.4044	0.3739	0.3466
26		0.5809	0.5488	0.5024	0.4629	0.4178	0.3880	0.3612
27		0.5907	0.5592	0.5139	0.4752	0.4308	0.4013	0.3743
28		0.5999	0.5691	0.5246	0.4864	0.4426	0.4138	0.3869
29		0.6087	0.5785	0.5348	0.4974	0.4544	0.4256	0.3991
30		0.6169	0.5874	0.5446	0.5080	0.4656	0.4369	0.4106
31		0.6248	0.5958	0.5539	0.5178	0.4762	0.4481	0.4218
32		0.6322	0.6039	0.5626	0.5270	0.4860	0.4585	0.4330
33		0.6394	0.6117	0.5712	0.5362	0.4959	0.4685	0.4431
34		0.6462	0.6191	0.5793	0.5448	0.5050	0.4777	0.4524
35		0.6528	0.6262	0.5871	0.5532	0.5141	0.4875	0.4627
36		0.6589	0.6328	0.5945	0.5612	0.5226	0.4961	0.4718
37		0.6649	0.6393	0.6016	0.5689	0.5306	0.5044	0.4805
38		0.6706	0.6455	0.6086	0.5763	0.5386	0.5129	0.4890
39		0.6761	0.6514	0.6150	0.5833	0.5463	0.5209	0.4974
40		0.6814	0.6572	0.6213	0.5901	0.5536	0.5284	0.5051
41		0.6865	0.6627	0.6274	0.5969	0.5608	0.5361	0.5125
42		0.6914	0.6680	0.6333	0.6032	0.5676	0.5429	0.5203
43		0.6961	0.6731	0.6390	0.6094	0.5743	0.5500	0.5274
44		0.7007	0.6781	0.6445	0.6152	0.5805	0.5566	0.5342
45		0.7051	0.6828	0.6497	0.6209	0.5869	0.5635	0.5415
46		0.7093	0.6874	0.6549	0.6264	0.5929	0.5697	0.5478
47		0.7134	0.6918	0.6599	0.6318	0.5986	0.5755	0.5539
48		0.7174	0.6961	0.6647	0.6370	0.6043	0.5815	0.5603
49		0.7213	0.7003	0.6693	0.6421	0.6098	0.5872	0.5663
50		0.7250	0.7044	0.6737	0.6469	0.6149	0.5930	0.5723
51		0.7286	0.7083	0.6781	0.6515	0.6202	0.5984	0.5777
52		0.7322	0.7121	0.6824	0.6561	0.6251	0.6036	0.5834
53		0.7355	0.7158	0.6865	0.6607	0.6300	0.6087	0.5882

continues

Table A8 (continued). Critical values for discordancy test N4 of three upper or lower outliers (k=3) in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
54		0.7388	0.7193	0.6904	0.6649	0.6346	0.6136	0.5938
55		0.7420	0.7228	0.6943	0.6691	0.6393	0.6184	0.5986
56		0.7451	0.7262	0.6981	0.6732	0.6437	0.6230	0.6035
57		0.7481	0.7295	0.7017	0.6772	0.6480	0.6273	0.6082
58		0.7511	0.7327	0.7053	0.6810	0.6523	0.6320	0.6128
59		0.7539	0.7359	0.7087	0.6848	0.6562	0.6363	0.6174
60		0.7567	0.7389	0.7120	0.6884	0.6603	0.6405	0.6220
61		0.7595	0.7418	0.7154	0.6919	0.6640	0.6445	0.6262
62		0.7622	0.7447	0.7185	0.6955	0.6679	0.6486	0.6304
63		0.7648	0.7474	0.7217	0.6989	0.6716	0.6524	0.6343
64		0.7672	0.7502	0.7247	0.7021	0.6753	0.6564	0.6387
65		0.7697	0.7528	0.7276	0.7054	0.6787	0.6601	0.6424
66		0.7721	0.7554	0.7306	0.7085	0.6820	0.6634	0.6460
67		0.7744	0.7580	0.7333	0.7116	0.6854	0.6670	0.6496
68		0.7768	0.7604	0.7361	0.7146	0.6888	0.6706	0.6538
69		0.7790	0.7629	0.7389	0.7174	0.6919	0.6740	0.6570
70		0.7811	0.7652	0.7415	0.7204	0.6951	0.6775	0.6614
71		0.7832	0.7675	0.7440	0.7232	0.6982	0.6807	0.6640
72		0.7853	0.7698	0.7465	0.7258	0.7010	0.6834	0.6672
73		0.7874	0.7720	0.7490	0.7286	0.7041	0.6867	0.6702
74		0.7894	0.7741	0.7514	0.7312	0.7068	0.6898	0.6734
75		0.7914	0.7763	0.7537	0.7337	0.7097	0.6929	0.6769
76		0.7933	0.7783	0.7560	0.7362	0.7123	0.6956	0.6798
77		0.7951	0.7803	0.7583	0.7387	0.7152	0.6987	0.6830
78		0.7969	0.7824	0.7605	0.7411	0.7177	0.7012	0.6857
79		0.7987	0.7843	0.7626	0.7434	0.7204	0.7039	0.6885
80		0.8005	0.7862	0.7647	0.7457	0.7228	0.7067	0.6915
81		0.8022	0.7880	0.7667	0.7479	0.7252	0.7092	0.6942
82		0.8039	0.7899	0.7688	0.7501	0.7278	0.7119	0.6969
83		0.8055	0.7916	0.7709	0.7524	0.7302	0.7144	0.6992
84		0.8072	0.7934	0.7728	0.7546	0.7325	0.7167	0.7017
85		0.8088	0.7951	0.7747	0.7565	0.7348	0.7194	0.7049
86		0.8103	0.7968	0.7766	0.7587	0.7370	0.7217	0.7070
87		0.8119	0.7985	0.7785	0.7606	0.7392	0.7240	0.7095
88		0.8133	0.8001	0.7803	0.7627	0.7413	0.7262	0.7118
89		0.8148	0.8017	0.7821	0.7646	0.7435	0.7287	0.7142
90		0.8163	0.8033	0.7838	0.7664	0.7456	0.7307	0.7167
91		0.8177	0.8048	0.7856	0.7683	0.7476	0.7328	0.7190
92		0.8191	0.8063	0.7872	0.7701	0.7496	0.7351	0.7212
93		0.8205	0.8079	0.7889	0.7720	0.7516	0.7371	0.7235
94		0.8219	0.8093	0.7905	0.7737	0.7534	0.7392	0.7258
95		0.8232	0.8107	0.7921	0.7754	0.7553	0.7410	0.7274
96		0.8245	0.8122	0.7937	0.7773	0.7574	0.7431	0.7297
97		0.8258	0.8136	0.7952	0.7789	0.7591	0.7451	0.7318
98		0.8270	0.8149	0.7968	0.7805	0.7610	0.7471	0.7339
99		0.8283	0.8163	0.7983	0.7822	0.7628	0.7489	0.7358
100		0.8295	0.8176	0.7997	0.7838	0.7645	0.7508	0.7380

CL: Confidence level (%); SL: Significance level (%); α: Significance level. The headers for CL or SL or α generally used for most applications are given in italic-bold face. The mean values of the standard error of the mean ($\bar{x}_{se}$) for these critical values ($\bar{x}$) are (respective % errors are also reported in parentheses): ~0.00008 (for α=0.30, 0.016%); ~0.00008 (for α=0.20, 0.019%); ~0.00010 (for α=0.10, 0.024%); ~0.00012 (for α=0.05, 0.034%); ~0.00016 (for α=0.02, 0.05%); ~0.00020 (for α=0.01, 0.07%); and ~0.00026 (for α=0.005, 0.09%).

Table A9. Critical values for discordancy test N4 of four upper or lower outliers (k=4) in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
3		---	---	---	---	---	---	---
4		---	---	---	---	---	---	---
5		---	---	---	---	---	---	---
6		---	---	---	---	---	---	---
7		---	---	---	---	---	---	---
8		0.0921	0.0650	0.0374	0.0221	0.0114	0.0070	0.0043
9		0.1349	0.1026	0.0666	0.0443	0.0265	0.0183	0.0127
10		0.1744	0.1389	0.0970	0.0695	0.0455	0.0335	0.0248
11		0.2105	0.1729	0.1274	0.0959	0.0669	0.0517	0.0401
12		0.2430	0.2046	0.1563	0.1221	0.0894	0.0713	0.0572
13		0.2729	0.2339	0.1840	0.1474	0.1119	0.0914	0.0754
14		0.2998	0.2608	0.2101	0.1723	0.1344	0.1121	0.0943
15		0.3247	0.2858	0.2346	0.1958	0.1564	0.1328	0.1132
16		0.3475	0.3090	0.2577	0.2179	0.1773	0.1525	0.1316
17		0.3684	0.3303	0.2793	0.2396	0.1975	0.1719	0.1503
18		0.3879	0.3504	0.2997	0.2595	0.2174	0.1914	0.1692
19		0.4057	0.3688	0.3186	0.2785	0.2357	0.2090	0.1857
20		0.4226	0.3863	0.3367	0.2969	0.2538	0.2264	0.2030
21		0.4382	0.4026	0.3536	0.3139	0.2708	0.2434	0.2198
22		0.4529	0.4181	0.3695	0.3299	0.2868	0.2590	0.2358
23		0.4667	0.4325	0.3848	0.3455	0.3027	0.2754	0.2507
24		0.4797	0.4461	0.3990	0.3604	0.3178	0.2902	0.2658
25		0.4918	0.4590	0.4126	0.3744	0.3319	0.3040	0.2793
26		0.5034	0.4710	0.4254	0.3878	0.3456	0.3182	0.2940
27		0.5143	0.4825	0.4377	0.4004	0.3588	0.3314	0.3069
28		0.5247	0.4935	0.4493	0.4126	0.3711	0.3439	0.3196
29		0.5345	0.5040	0.4605	0.4242	0.3833	0.3561	0.3314
30		0.5439	0.5138	0.4712	0.4353	0.3947	0.3680	0.3438
31		0.5527	0.5232	0.4813	0.4461	0.4060	0.3795	0.3553
32		0.5612	0.5322	0.4909	0.4559	0.4163	0.3905	0.3664
33		0.5695	0.5410	0.5002	0.4657	0.4270	0.4008	0.3767
34		0.5772	0.5493	0.5091	0.4750	0.4361	0.4104	0.3868
35		0.5847	0.5573	0.5177	0.4841	0.4459	0.4204	0.3973
36		0.5918	0.5647	0.5260	0.4927	0.4551	0.4295	0.4066
37		0.5986	0.5721	0.5338	0.5011	0.4638	0.4385	0.4155
38		0.6052	0.5791	0.5414	0.5092	0.4723	0.4471	0.4243
39		0.6116	0.5858	0.5486	0.5169	0.4803	0.4559	0.4331
40		0.6176	0.5923	0.5557	0.5243	0.4881	0.4640	0.4416
41		0.6235	0.5985	0.5625	0.5317	0.4960	0.4719	0.4494
42		0.6291	0.6046	0.5691	0.5386	0.5033	0.4793	0.4577
43		0.6346	0.6104	0.5754	0.5454	0.5106	0.4869	0.4649
44		0.6399	0.6161	0.5815	0.5517	0.5172	0.4941	0.4724
45		0.6449	0.6215	0.5874	0.5583	0.5243	0.5014	0.4797
46		0.6498	0.6268	0.5931	0.5643	0.5307	0.5080	0.4871
47		0.6546	0.6318	0.5987	0.5701	0.5371	0.5144	0.4934
48		0.6592	0.6367	0.6042	0.5759	0.5432	0.5208	0.4999
49		0.6637	0.6416	0.6094	0.5816	0.5492	0.5269	0.5065
50		0.6681	0.6462	0.6143	0.5869	0.5548	0.5330	0.5128
51		0.6722	0.6507	0.6193	0.5922	0.5607	0.5389	0.5187
52		0.6764	0.6552	0.6242	0.5972	0.5660	0.5447	0.5245
53		0.6803	0.6594	0.6288	0.6023	0.5712	0.5500	0.5302

continues

Table A9 (continued). Critical values for discordancy test N4 of four upper or lower outliers (k=4) in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
54		0.6842	0.6635	0.6333	0.6071	0.5764	0.5556	0.5364
55		0.6879	0.6675	0.6377	0.6117	0.5816	0.5606	0.5411
56		0.6915	0.6713	0.6419	0.6163	0.5864	0.5657	0.5467
57		0.6951	0.6752	0.6461	0.6208	0.5911	0.5705	0.5514
58		0.6985	0.6789	0.6500	0.6252	0.5959	0.5756	0.5567
59		0.7018	0.6825	0.6540	0.6293	0.6003	0.5803	0.5613
60		0.7051	0.6860	0.6579	0.6333	0.6049	0.5851	0.5667
61		0.7084	0.6894	0.6615	0.6374	0.6088	0.5894	0.5711
62		0.7115	0.6927	0.6653	0.6413	0.6131	0.5939	0.5759
63		0.7145	0.6960	0.6688	0.6451	0.6174	0.5983	0.5801
64		0.7174	0.6991	0.6723	0.6489	0.6214	0.6025	0.5848
65		0.7204	0.7022	0.6756	0.6526	0.6253	0.6063	0.5886
66		0.7231	0.7052	0.6790	0.6560	0.6291	0.6103	0.5928
67		0.7260	0.7083	0.6823	0.6595	0.6327	0.6141	0.5970
68		0.7286	0.7111	0.6854	0.6629	0.6364	0.6182	0.6009
69		0.7313	0.7139	0.6884	0.6661	0.6400	0.6219	0.6048
70		0.7338	0.7167	0.6915	0.6695	0.6436	0.6256	0.6089
71		0.7363	0.7194	0.6945	0.6727	0.6469	0.6292	0.6123
72		0.7388	0.7220	0.6973	0.6757	0.6501	0.6324	0.6156
73		0.7412	0.7246	0.7001	0.6788	0.6535	0.6359	0.6193
74		0.7435	0.7271	0.7029	0.6818	0.6568	0.6395	0.6230
75		0.7459	0.7296	0.7055	0.6846	0.6599	0.6426	0.6265
76		0.7481	0.7320	0.7082	0.6875	0.6630	0.6458	0.6297
77		0.7503	0.7344	0.7108	0.6904	0.6660	0.6491	0.6332
78		0.7525	0.7367	0.7135	0.6931	0.6690	0.6522	0.6364
79		0.7546	0.7389	0.7158	0.6957	0.6717	0.6551	0.6394
80		0.7567	0.7412	0.7183	0.6983	0.6747	0.6582	0.6426
81		0.7587	0.7433	0.7207	0.7009	0.6773	0.6610	0.6457
82		0.7607	0.7455	0.7231	0.7035	0.6803	0.6641	0.6487
83		0.7626	0.7476	0.7255	0.7060	0.6829	0.6668	0.6516
84		0.7646	0.7497	0.7277	0.7084	0.6855	0.6693	0.6543
85		0.7665	0.7517	0.7298	0.7108	0.6882	0.6724	0.6574
86		0.7683	0.7537	0.7320	0.7132	0.6906	0.6750	0.6601
87		0.7702	0.7557	0.7342	0.7155	0.6931	0.6775	0.6628
88		0.7719	0.7576	0.7364	0.7177	0.6956	0.6801	0.6655
89		0.7737	0.7594	0.7384	0.7199	0.6981	0.6828	0.6682
90		0.7755	0.7613	0.7404	0.7221	0.7004	0.6852	0.6708
91		0.7771	0.7631	0.7425	0.7243	0.7026	0.6876	0.6733
92		0.7788	0.7649	0.7444	0.7264	0.7050	0.6900	0.6758
93		0.7804	0.7667	0.7464	0.7285	0.7072	0.6924	0.6783
94		0.7820	0.7684	0.7481	0.7304	0.7094	0.6946	0.6809
95		0.7836	0.7701	0.7501	0.7325	0.7115	0.6968	0.6829
96		0.7852	0.7718	0.7519	0.7345	0.7137	0.6993	0.6854
97		0.7867	0.7734	0.7537	0.7364	0.7158	0.7012	0.6877
98		0.7882	0.7750	0.7555	0.7383	0.7179	0.7037	0.6901
99		0.7897	0.7766	0.7572	0.7402	0.7198	0.7057	0.6922
100		0.7911	0.7782	0.7589	0.7420	0.7218	0.7078	0.6944

CL: Confidence level (%); SL: Significance level (%); α: Significance level. The headers for CL or SL or α generally used for most applications are given in italic-bold face. The mean values of the standard error of the mean ($\bar{x}_{se}$) for these critical values ($\bar{x}$) are (respective % errors are also reported in parentheses): ~0.00007 (for α=0.30, 0.017%); ~0.00008 (for α=0.20, 0.020%); ~0.00009 (for α=0.10, 0.025%); ~0.00012 (for α=0.05, 0.035%); ~0.00016 (for α=0.02, 0.05%); ~0.00020 (for α=0.01, 0.07%); and ~0.00025 (for α=0.005, 0.09%).

Table A10. Critical values for discordancy test N5 of two upper or lower outliers (k=2) in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
3		---	---	---	---	---	---	---
4		0.0186	0.0079	0.0019	0.0005	0.0001	0.0000	0.0000
5		0.0870	0.0554	0.0264	0.0129	0.0050	0.0025	0.0012
6		0.1575	0.1158	0.0698	0.0425	0.0224	0.0141	0.0087
7		0.2185	0.1729	0.1178	0.0811	0.0500	0.0348	0.0245
8		0.2708	0.2242	0.1644	0.1217	0.0825	0.0617	0.0462
9		0.3150	0.2688	0.2069	0.1606	0.1158	0.0909	0.0715
10		0.3536	0.3082	0.2463	0.1981	0.1497	0.1211	0.0985
11		0.3871	0.3430	0.2817	0.2330	0.1819	0.1519	0.1263
12		0.4168	0.3744	0.3141	0.2650	0.2126	0.1806	0.1536
13		0.4432	0.4019	0.3428	0.2939	0.2407	0.2078	0.1799
14		0.4668	0.4268	0.3692	0.3208	0.2673	0.2341	0.2053
15		0.4881	0.4496	0.3935	0.3457	0.2925	0.2588	0.2286
16		0.5076	0.4705	0.4158	0.3695	0.3167	0.2830	0.2523
17		0.5253	0.4893	0.4362	0.3905	0.3392	0.3052	0.2749
18		0.5417	0.5068	0.4549	0.4100	0.3592	0.3253	0.2949
19		0.5567	0.5230	0.4724	0.4285	0.3783	0.3449	0.3146
20		0.5706	0.5380	0.4890	0.4462	0.3961	0.3631	0.3325
21		0.5836	0.5519	0.5041	0.4620	0.4132	0.3807	0.3507
22		0.5958	0.5648	0.5182	0.4771	0.4291	0.3972	0.3676
23		0.6070	0.5768	0.5314	0.4913	0.4443	0.4125	0.3828
24		0.6175	0.5884	0.5444	0.5054	0.4596	0.4278	0.3997
25		0.6275	0.5992	0.5560	0.5174	0.4721	0.4415	0.4128
26		0.6369	0.6092	0.5672	0.5299	0.4855	0.4553	0.4270
27		0.6455	0.6186	0.5777	0.5408	0.4973	0.4672	0.4399
28		0.6542	0.6279	0.5877	0.5519	0.5086	0.4791	0.4520
29		0.6620	0.6364	0.5972	0.5618	0.5199	0.4909	0.4643
30		0.6694	0.6443	0.6060	0.5717	0.5304	0.5014	0.4749
31		0.6766	0.6520	0.6146	0.5810	0.5405	0.5122	0.4858
32		0.6834	0.6593	0.6228	0.5895	0.5497	0.5221	0.4963
33		0.6898	0.6662	0.6303	0.5978	0.5586	0.5315	0.5062
34		0.6960	0.6728	0.6377	0.6060	0.5674	0.5409	0.5157
35		0.7020	0.6794	0.6448	0.6135	0.5755	0.5488	0.5241
36		0.7076	0.6853	0.6516	0.6210	0.5838	0.5575	0.5329
37		0.7128	0.6913	0.6580	0.6280	0.5916	0.5660	0.5413
38		0.7180	0.6968	0.6644	0.6346	0.5990	0.5736	0.5498
39		0.7230	0.7022	0.6703	0.6414	0.6062	0.5818	0.5582
40		0.7278	0.7074	0.6760	0.6476	0.6127	0.5879	0.5649
41		0.7324	0.7122	0.6814	0.6534	0.6194	0.5948	0.5723
42		0.7367	0.7171	0.6867	0.6592	0.6258	0.6021	0.5797
43		0.7410	0.7217	0.6919	0.6647	0.6316	0.6080	0.5862
44		0.7449	0.7260	0.6968	0.6700	0.6378	0.6145	0.5921
45		0.7490	0.7303	0.7016	0.6755	0.6434	0.6202	0.5987
46		0.7527	0.7343	0.7060	0.6802	0.6486	0.6260	0.6048
47		0.7565	0.7384	0.7106	0.6852	0.6541	0.6320	0.6110
48		0.7600	0.7422	0.7149	0.6898	0.6594	0.6372	0.6167
49		0.7635	0.7461	0.7191	0.6945	0.6641	0.6427	0.6223
50		0.7669	0.7497	0.7230	0.6988	0.6689	0.6481	0.6275
51		0.7700	0.7530	0.7270	0.7031	0.6736	0.6527	0.6329
52		0.7732	0.7565	0.7308	0.7072	0.6782	0.6576	0.6375
53		0.7762	0.7598	0.7345	0.7112	0.6826	0.6622	0.6426

continues

Table A10 (continued). Critical values for discordancy test N5 of two upper or lower outliers (k=2) in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
54		0.7791	0.7628	0.7379	0.7151	0.6872	0.6669	0.6474
55		0.7820	0.7661	0.7415	0.7190	0.6912	0.6715	0.6522
56		0.7848	0.7691	0.7448	0.7225	0.6953	0.6756	0.6567
57		0.7875	0.7720	0.7480	0.7261	0.6991	0.6797	0.6610
58		0.7901	0.7748	0.7512	0.7297	0.7029	0.6837	0.6651
59		0.7927	0.7775	0.7542	0.7329	0.7065	0.6875	0.6692
60		0.7952	0.7802	0.7572	0.7363	0.7103	0.6915	0.6737
61		0.7975	0.7828	0.7601	0.7393	0.7136	0.6951	0.6774
62		0.7999	0.7853	0.7631	0.7425	0.7171	0.6990	0.6815
63		0.8022	0.7878	0.7657	0.7455	0.7205	0.7026	0.6853
64		0.8044	0.7902	0.7684	0.7484	0.7238	0.7059	0.6886
65		0.8065	0.7926	0.7711	0.7513	0.7268	0.7094	0.6925
66		0.8086	0.7949	0.7736	0.7540	0.7298	0.7124	0.6956
67		0.8107	0.7971	0.7761	0.7566	0.7327	0.7156	0.6990
68		0.8128	0.7993	0.7785	0.7594	0.7357	0.7185	0.7021
69		0.8148	0.8014	0.7809	0.7620	0.7388	0.7218	0.7058
70		0.8167	0.8035	0.7832	0.7646	0.7414	0.7249	0.7091
71		0.8186	0.8056	0.7854	0.7670	0.7440	0.7278	0.7121
72		0.8204	0.8076	0.7878	0.7695	0.7469	0.7305	0.7148
73		0.8222	0.8095	0.7899	0.7717	0.7493	0.7331	0.7177
74		0.8239	0.8113	0.7920	0.7741	0.7519	0.7360	0.7203
75		0.8256	0.8131	0.7940	0.7764	0.7542	0.7385	0.7233
76		0.8273	0.8150	0.7960	0.7785	0.7569	0.7413	0.7263
77		0.8289	0.8168	0.7979	0.7807	0.7593	0.7438	0.7288
78		0.8306	0.8185	0.7999	0.7829	0.7614	0.7465	0.7314
79		0.8321	0.8202	0.8018	0.7849	0.7639	0.7488	0.7340
80		0.8337	0.8219	0.8037	0.7869	0.7663	0.7513	0.7369
81		0.8352	0.8235	0.8055	0.7888	0.7682	0.7535	0.7390
82		0.8366	0.8251	0.8073	0.7908	0.7703	0.7557	0.7415
83		0.8381	0.8267	0.8091	0.7928	0.7727	0.7582	0.7440
84		0.8395	0.8282	0.8107	0.7946	0.7745	0.7600	0.7461
85		0.8409	0.8297	0.8124	0.7964	0.7764	0.7620	0.7482
86		0.8423	0.8312	0.8141	0.7983	0.7786	0.7644	0.7507
87		0.8436	0.8326	0.8157	0.8000	0.7805	0.7666	0.7532
88		0.8450	0.8341	0.8172	0.8016	0.7824	0.7682	0.7551
89		0.8463	0.8355	0.8189	0.8034	0.7844	0.7706	0.7570
90		0.8475	0.8369	0.8203	0.8051	0.7861	0.7726	0.7597
91		0.8488	0.8382	0.8218	0.8066	0.7878	0.7743	0.7616
92		0.8500	0.8395	0.8233	0.8083	0.7897	0.7762	0.7629
93		0.8512	0.8408	0.8248	0.8099	0.7914	0.7782	0.7653
94		0.8523	0.8420	0.8261	0.8115	0.7932	0.7801	0.7671
95		0.8535	0.8434	0.8276	0.8129	0.7947	0.7816	0.7689
96		0.8547	0.8445	0.8289	0.8144	0.7966	0.7834	0.7708
97		0.8558	0.8458	0.8303	0.8159	0.7982	0.7853	0.7728
98		0.8568	0.8469	0.8316	0.8173	0.7996	0.7869	0.7745
99		0.8580	0.8481	0.8329	0.8189	0.8014	0.7887	0.7764
100		0.8590	0.8493	0.8341	0.8202	0.8028	0.7905	0.7782

CL: Confidence level (%); SL: Significance level (%); α: Significance level. The headers for CL or SL or α generally used for most applications are given in italic-bold face. The mean values of the standard error of the mean ($\bar{x}_{se}$) for these critical values ($\bar{x}$) are (respective % errors are also reported in parentheses): ~0.00008 (for α=0.30, 0.020%); ~0.00009 (for α=0.20, 0.024%); ~0.00011 (for α=0.10, 0.032%); ~0.00015 (for α=0.05, 0.05%); ~0.00023 (for α=0.02, 0.08%); ~0.00031 (for α=0.01, 0.10%); and ~0.00042 (for α=0.005, 0.14%).

Table A11. Critical values for discordancy test N6 of upper and lower outlier pair (k=2) in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
3		1.9754	1.9891	1.9973	1.9993	1.9999	2.0000	2.0000
4		2.3270	2.3678	2.4087	2.4291	2.4413	2.4454	2.4474
5		2.5846	2.6428	2.7118	2.7550	2.7886	2.8033	2.8126
6		2.7900	2.8594	2.9492	3.0121	3.0675	3.0954	3.1150
7		2.9605	3.0383	3.1436	3.2226	3.2974	3.3382	3.3690
8		3.1037	3.1903	3.3080	3.3998	3.4912	3.5439	3.5848
9		3.2257	3.3211	3.4496	3.5520	3.6580	3.7206	3.7718
10		3.3318	3.4350	3.5740	3.6854	3.8035	3.8752	3.9350
11		3.4251	3.5351	3.6839	3.8030	3.9316	4.0113	4.0791
12		3.5093	3.6250	3.7824	3.9094	4.0469	4.1337	4.2085
13		3.5855	3.7061	3.8715	4.0053	4.1513	4.2446	4.3260
14		3.6550	3.7797	3.9518	4.0916	4.2450	4.3443	4.4303
15		3.7189	3.8470	4.0255	4.1710	4.3316	4.4357	4.5283
16		3.7780	3.9093	4.0933	4.2443	4.4113	4.5197	4.6166
17		3.8335	3.9675	4.1561	4.3119	4.4844	4.5971	4.6969
18		3.8847	4.0214	4.2144	4.3745	4.5528	4.6696	4.7741
19		3.9328	4.0719	4.2694	4.4329	4.6164	4.7371	4.8447
20		3.9783	4.1195	4.3201	4.4873	4.6755	4.7995	4.9111
21		4.0211	4.1638	4.3678	4.5389	4.7317	4.8593	4.9736
22		4.0616	4.2067	4.4138	4.5875	4.7833	4.9128	5.0307
23		4.1003	4.2471	4.4569	4.6335	4.8334	4.9659	5.0863
24		4.1368	4.2847	4.4968	4.6763	4.8799	5.0149	5.1377
25		4.1720	4.3213	4.5359	4.7170	4.9235	5.0617	5.1870
26		4.2049	4.3554	4.5718	4.7562	4.9659	5.1068	5.2339
27		4.2374	4.3890	4.6077	4.7938	5.0061	5.1490	5.2787
28		4.2675	4.4200	4.6408	4.8286	5.0435	5.1874	5.3194
29		4.2967	4.4508	4.6732	4.8630	5.0807	5.2267	5.3605
30		4.3253	4.4800	4.7036	4.8951	5.1151	5.2621	5.3978
31		4.3524	4.5081	4.7341	4.9270	5.1485	5.2993	5.4382
32		4.3787	4.5350	4.7620	4.9563	5.1810	5.3331	5.4724
33		4.4038	4.5611	4.7897	4.9857	5.2120	5.3652	5.5039
34		4.4284	4.5863	4.8161	5.0128	5.2407	5.3955	5.5371
35		4.4519	4.6107	4.8418	5.0403	5.2707	5.4265	5.5712
36		4.4750	4.6343	4.8665	5.0662	5.2967	5.4543	5.5996
37		4.4972	4.6567	4.8907	5.0913	5.3235	5.4811	5.6276
38		4.5180	4.6787	4.9133	5.1154	5.3493	5.5081	5.6563
39		4.5389	4.7000	4.9357	5.1387	5.3735	5.5336	5.6823
40		4.5593	4.7210	4.9574	5.1611	5.3983	5.5611	5.7105
41		4.5793	4.7415	4.9782	5.1833	5.4216	5.5844	5.7340
42		4.5983	4.7610	4.9990	5.2045	5.4431	5.6058	5.7573
43		4.6168	4.7797	5.0186	5.2259	5.4659	5.6303	5.7835
44		4.6351	4.7986	5.0385	5.2455	5.4866	5.6519	5.8069
45		4.6524	4.8163	5.0565	5.2654	5.5076	5.6731	5.8283
46		4.6700	4.8344	5.0754	5.2840	5.5280	5.6950	5.8507
47		4.6868	4.8517	5.0931	5.3032	5.5485	5.7147	5.8709
48		4.7033	4.8687	5.1104	5.3208	5.5660	5.7347	5.8902
49		4.7191	4.8846	5.1278	5.3385	5.5848	5.7540	5.9100
50		4.7344	4.9001	5.1441	5.3557	5.6032	5.7729	5.9300
51		4.7500	4.9165	5.1607	5.3731	5.6214	5.7922	5.9513
52		4.7649	4.9313	5.1760	5.3895	5.6388	5.8102	5.9680
53		4.7796	4.9464	5.1921	5.4059	5.6547	5.8259	5.9855

continues

Table A11 (continued). Critical values for discordancy test N6 of upper and lower outlier pair (k=2) in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
54		4.7939	4.9612	5.2067	5.4210	5.6709	5.8434	6.0045
55		4.8078	4.9752	5.2214	5.4360	5.6874	5.8603	6.0212
56		4.8216	4.9891	5.2360	5.4507	5.7017	5.8742	6.0364
57		4.8351	5.0031	5.2508	5.4656	5.7179	5.8911	6.0531
58		4.8483	5.0165	5.2640	5.4800	5.7335	5.9082	6.0702
59		4.8611	5.0295	5.2779	5.4936	5.7473	5.9232	6.0861
60		4.8739	5.0427	5.2914	5.5079	5.7615	5.9376	6.1016
61		4.8863	5.0553	5.3039	5.5215	5.7773	5.9529	6.1166
62		4.8987	5.0677	5.3168	5.5345	5.7900	5.9663	6.1306
63		4.9106	5.0799	5.3292	5.5467	5.8023	5.9797	6.1448
64		4.9227	5.0923	5.3417	5.5600	5.8173	5.9944	6.1603
65		4.9340	5.1035	5.3534	5.5721	5.8300	6.0066	6.1712
66		4.9456	5.1156	5.3662	5.5849	5.8425	6.0199	6.1857
67		4.9567	5.1271	5.3779	5.5972	5.8560	6.0334	6.2015
68		4.9679	5.1383	5.3894	5.6076	5.8675	6.0469	6.2143
69		4.9784	5.1492	5.4005	5.6199	5.8786	6.0576	6.2242
70		4.9893	5.1598	5.4113	5.6318	5.8920	6.0708	6.2375
71		4.9997	5.1707	5.4227	5.6434	5.9032	6.0826	6.2504
72		5.0101	5.1808	5.4332	5.6539	5.9146	6.0945	6.2632
73		5.0203	5.1917	5.4434	5.6660	5.9266	6.1059	6.2740
74		5.0301	5.2016	5.4538	5.6752	5.9360	6.1173	6.2854
75		5.0400	5.2112	5.4645	5.6861	5.9481	6.1290	6.2972
76		5.0494	5.2211	5.4747	5.6958	5.9582	6.1396	6.3089
77		5.0593	5.2310	5.4841	5.7061	5.9689	6.1511	6.3214
78		5.0685	5.2405	5.4938	5.7154	5.9793	6.1598	6.3299
79		5.0778	5.2498	5.5038	5.7261	5.9891	6.1709	6.3400
80		5.0871	5.2588	5.5131	5.7359	5.9988	6.1816	6.3517
81		5.0962	5.2681	5.5226	5.7461	6.0098	6.1919	6.3628
82		5.1054	5.2776	5.5319	5.7549	6.0194	6.2018	6.3718
83		5.1143	5.2865	5.5405	5.7641	6.0280	6.2116	6.3814
84		5.1228	5.2947	5.5495	5.7735	6.0384	6.2222	6.3934
85		5.1314	5.3038	5.5588	5.7820	6.0480	6.2303	6.4020
86		5.1397	5.3123	5.5669	5.7911	6.0562	6.2400	6.4119
87		5.1478	5.3206	5.5763	5.8001	6.0654	6.2501	6.4220
88		5.1560	5.3288	5.5847	5.8095	6.0757	6.2605	6.4332
89		5.1644	5.3372	5.5932	5.8183	6.0843	6.2682	6.4429
90		5.1724	5.3451	5.6012	5.8257	6.0925	6.2773	6.4490
91		5.1799	5.3533	5.6093	5.8343	6.1014	6.2859	6.4606
92		5.1881	5.3609	5.6175	5.8430	6.1098	6.2945	6.4676
93		5.1956	5.3692	5.6256	5.8505	6.1184	6.3042	6.4775
94		5.2034	5.3770	5.6334	5.8589	6.1262	6.3114	6.4858
95		5.2110	5.3843	5.6408	5.8661	6.1336	6.3194	6.4938
96		5.2184	5.3918	5.6487	5.8741	6.1416	6.3277	6.5026
97		5.2258	5.3992	5.6559	5.8818	6.1495	6.3361	6.5131
98		5.2329	5.4066	5.6635	5.8897	6.1573	6.3429	6.5189
99		5.2399	5.4135	5.6706	5.8970	6.1652	6.3514	6.5252
100		5.2473	5.4209	5.6785	5.9043	6.1727	6.3593	6.5345

CL: Confidence level (%); SL: Significance level (%); α: Significance level. The headers for CL or SL or α generally used for most applications are given in italic-bold face. The mean values of the standard error of the mean ($\bar{x}_{se}$) for these critical values ($\bar{x}$) are (respective % errors are also reported in parentheses): ~0.00020 (for α=0.30, 0.0044%); ~0.00021 (for α=0.20, 0.0045%); ~0.00027 (for α=0.10, 0.005%); ~0.00035 (for α=0.05, 0.007%); ~0.0005 (for α=0.02, 0.009%); ~0.0006 (for α=0.01, 0.011%); and ~0.0008 (for α=0.005, 0.014%).

Table A12. Critical values for discordancy test N8 of an extreme outlier in a normal sample.

n	CL	70%	80%	90%	**95%**	98%	**99%**	99.5%
	SL	30%	20%	10%	**5%**	2%	**1%**	0.5%
	α	0.30	0.20	0.10	**0.05**	0.02	**0.01**	0.005
3		---	---	---	---	---	---	---
4		0.6142	0.6787	0.7653	0.8296	0.8888	0.9201	0.9426
5		0.4990	0.5583	0.6425	0.7106	0.7820	0.8238	0.8579
6		0.4297	0.4844	0.5625	0.6272	0.6983	0.7428	0.7822
7		0.3830	0.4341	0.5077	0.5690	0.6372	0.6811	0.7197
8		0.3495	0.3976	0.4669	0.5258	0.5907	0.6334	0.6714
9		0.3244	0.3702	0.4362	0.4922	0.5553	0.5953	0.6322
10		0.3045	0.3487	0.4118	0.4653	0.5261	0.5659	0.6005
11		0.2884	0.3308	0.3920	0.4435	0.5023	0.5413	0.5758
12		0.2751	0.3163	0.3754	0.4253	0.4823	0.5205	0.5533
13		0.2639	0.3038	0.3614	0.4105	0.4662	0.5032	0.5356
14		0.2544	0.2931	0.3495	0.3972	0.4516	0.4874	0.5203
15		0.2457	0.2839	0.3388	0.3854	0.4386	0.4741	0.5065
16		0.2385	0.2755	0.3291	0.3750	0.4271	0.4614	0.4929
17		0.2318	0.2682	0.3208	0.3658	0.4166	0.4509	0.4812
18		0.2258	0.2615	0.3134	0.3579	0.4076	0.4418	0.4721
19		0.2205	0.2555	0.3069	0.3503	0.3999	0.4331	0.4628
20		0.2158	0.2503	0.3004	0.3434	0.3921	0.4255	0.4551
21		0.2112	0.2453	0.2948	0.3371	0.3853	0.4177	0.4473
22		0.2074	0.2408	0.2898	0.3318	0.3791	0.4111	0.4407
23		0.2035	0.2369	0.2850	0.3262	0.3739	0.4052	0.4338
24		0.2002	0.2328	0.2804	0.3212	0.3678	0.3991	0.4268
25		0.1968	0.2292	0.2762	0.3169	0.3629	0.3938	0.4222
26		0.1940	0.2260	0.2726	0.3126	0.3583	0.3888	0.4167
27		0.1911	0.2228	0.2689	0.3087	0.3542	0.3845	0.4118
28		0.1885	0.2197	0.2655	0.3048	0.3500	0.3804	0.4073
29		0.1859	0.2171	0.2622	0.3013	0.3459	0.3756	0.4030
30		0.1837	0.2144	0.2594	0.2979	0.3423	0.3717	0.3987
31		0.1815	0.2120	0.2566	0.2949	0.3388	0.3684	0.3946
32		0.1794	0.2097	0.2537	0.2919	0.3356	0.3648	0.3914
33		0.1776	0.2076	0.2513	0.2889	0.3326	0.3616	0.3876
34		0.1757	0.2054	0.2488	0.2863	0.3296	0.3586	0.3846
35		0.1739	0.2033	0.2466	0.2837	0.3268	0.3554	0.3810
36		0.1722	0.2015	0.2445	0.2814	0.3241	0.3525	0.3783
37		0.1706	0.1997	0.2423	0.2789	0.3215	0.3499	0.3757
38		0.1692	0.1981	0.2405	0.2769	0.3188	0.3468	0.3728
39		0.1677	0.1962	0.2381	0.2743	0.3163	0.3441	0.3701
40		0.1660	0.1946	0.2364	0.2726	0.3142	0.3424	0.3679
41		0.1648	0.1931	0.2347	0.2707	0.3125	0.3406	0.3659
42		0.1636	0.1918	0.2331	0.2687	0.3104	0.3380	0.3638
43		0.1623	0.1902	0.2315	0.2669	0.3082	0.3356	0.3607
44		0.1612	0.1891	0.2298	0.2651	0.3062	0.3333	0.3584
45		0.1601	0.1877	0.2284	0.2637	0.3048	0.3321	0.3567
46		0.1589	0.1864	0.2270	0.2623	0.3029	0.3300	0.3550
47		0.1577	0.1852	0.2253	0.2603	0.3008	0.3280	0.3525
48		0.1567	0.1839	0.2240	0.2587	0.2991	0.3261	0.3510
49		0.1557	0.1828	0.2226	0.2573	0.2975	0.3242	0.3487
50		0.1547	0.1817	0.2213	0.2560	0.2959	0.3225	0.3467
51		0.1538	0.1807	0.2203	0.2548	0.2945	0.3213	0.3458
52		0.1528	0.1795	0.2188	0.2530	0.2928	0.3193	0.3436
53		0.1521	0.1787	0.2179	0.2520	0.2917	0.3181	0.3423

continues

Table A12 (continued). Critical values for discordancy test N8 of an extreme outlier in a normal sample.

n	CL SL α	70% 30% 0.30	80% 20% 0.20	90% 10% 0.10	**95%** **5%** **0.05**	98% 2% 0.02	**99%** **1%** **0.01**	99.5% 0.5% 0.005
54		0.1511	0.1776	0.2166	0.2505	0.2898	0.3164	0.3409
55		0.1502	0.1765	0.2155	0.2494	0.2882	0.3144	0.3387
56		0.1495	0.1757	0.2143	0.2481	0.2876	0.3137	0.3381
57		0.1486	0.1748	0.2133	0.2471	0.2860	0.3124	0.3362
58		0.1479	0.1740	0.2125	0.2460	0.2849	0.3107	0.3346
59		0.1470	0.1728	0.2113	0.2447	0.2833	0.3094	0.3332
60		0.1462	0.1721	0.2103	0.2433	0.2817	0.3078	0.3317
61		0.1456	0.1713	0.2094	0.2423	0.2811	0.3068	0.3307
62		0.1448	0.1705	0.2083	0.2414	0.2798	0.3061	0.3293
63		0.1441	0.1696	0.2073	0.2403	0.2788	0.3045	0.3281
64		0.1435	0.1691	0.2067	0.2395	0.2776	0.3033	0.3269
65		0.1430	0.1684	0.2058	0.2386	0.2767	0.3023	0.3256
66		0.1423	0.1676	0.2049	0.2376	0.2755	0.3008	0.3248
67		0.1416	0.1668	0.2043	0.2368	0.2746	0.3002	0.3235
68		0.1410	0.1662	0.2034	0.2359	0.2736	0.2991	0.3225
69		0.1404	0.1654	0.2026	0.2351	0.2725	0.2983	0.3218
70		0.1399	0.1648	0.2019	0.2341	0.2717	0.2969	0.3201
71		0.1393	0.1642	0.2011	0.2335	0.2712	0.2963	0.3190
72		0.1388	0.1636	0.2002	0.2323	0.2696	0.2949	0.3184
73		0.1382	0.1630	0.1997	0.2315	0.2687	0.2936	0.3166
74		0.1377	0.1623	0.1988	0.2309	0.2680	0.2936	0.3166
75		0.1372	0.1619	0.1983	0.2304	0.2672	0.2921	0.3158
76		0.1369	0.1614	0.1977	0.2294	0.2662	0.2915	0.3148
77		0.1363	0.1608	0.1970	0.2289	0.2657	0.2906	0.3131
78		0.1357	0.1601	0.1963	0.2278	0.2647	0.2898	0.3124
79		0.1353	0.1595	0.1955	0.2272	0.2640	0.2888	0.3120
80		0.1348	0.1590	0.1949	0.2266	0.2633	0.2882	0.3109
81		0.1343	0.1584	0.1942	0.2257	0.2625	0.2875	0.3100
82		0.1339	0.1581	0.1938	0.2252	0.2619	0.2866	0.3089
83		0.1334	0.1575	0.1932	0.2245	0.2611	0.2857	0.3081
84		0.1331	0.1571	0.1925	0.2238	0.2602	0.2846	0.3069
85		0.1325	0.1564	0.1919	0.2231	0.2593	0.2843	0.3071
86		0.1321	0.1561	0.1915	0.2225	0.2588	0.2835	0.3058
87		0.1316	0.1555	0.1908	0.2219	0.2580	0.2827	0.3049
88		0.1312	0.1551	0.1904	0.2213	0.2571	0.2817	0.3043
89		0.1309	0.1547	0.1898	0.2208	0.2572	0.2816	0.3039
90		0.1305	0.1542	0.1893	0.2202	0.2560	0.2802	0.3025
91		0.1300	0.1536	0.1887	0.2196	0.2556	0.2800	0.3021
92		0.1296	0.1532	0.1884	0.2189	0.2547	0.2792	0.3013
93		0.1294	0.1528	0.1878	0.2184	0.2542	0.2784	0.3004
94		0.1290	0.1525	0.1874	0.2180	0.2539	0.2780	0.3003
95		0.1286	0.1520	0.1867	0.2173	0.2528	0.2769	0.2991
96		0.1282	0.1517	0.1864	0.2168	0.2524	0.2763	0.2986
97		0.1279	0.1513	0.1859	0.2162	0.2518	0.2760	0.2975
98		0.1276	0.1508	0.1853	0.2158	0.2514	0.2754	0.2972
99		0.1271	0.1504	0.1849	0.2153	0.2505	0.2745	0.2963
100		0.1269	0.1501	0.1845	0.2147	0.2501	0.2743	0.2966

CL: Confidence level (%); SL: Significance level (%); α: Significance level. The headers for CL or SL or α generally used for most applications are given in italic-bold face. The mean values of the standard error of the mean ($\bar{x}_{se}$) for these critical values ($\bar{x}$) are (respective % errors are also reported in parentheses): ~0.00012 (for α=0.30, 0.07%); ~0.00013 (for α=0.20, 0.06%); ~0.00015 (for α=0.10, 0.06%); ~0.00020 (for α=0.05, 0.07%); ~0.00029 (for α=0.02, 0.09%); ~0.00036 (for α=0.01, 0.10%); and ~0.0005 (for α=0.005, 0.13%).

Table A13. Critical values for skewness discordancy test N14 of an extreme outlier in a normal sample.

n	CL SL α	70% 30% 0.30	80% 20% 0.20	90% 10% 0.10	***95%*** ***5%*** ***0.05***	98% 2% 0.02	***99%*** ***1%*** ***0.01***	99.5% 0.5% 0.005
3		---	---	---	---	---	---	---
4		---	---	---	---	---	---	---
5		0.3294	0.5165	0.8215	1.0490	1.2459	1.3375	1.3963
6		0.3222	0.5202	0.7954	1.0432	1.2928	1.4303	1.5324
7		0.3116	0.5066	0.7824	1.0187	1.2917	1.4564	1.5878
8		0.3043	0.4927	0.7651	0.9982	1.2723	1.4525	1.6050
9		0.2959	0.4804	0.7459	0.9765	1.2464	1.4312	1.5966
10		0.2887	0.4679	0.7273	0.9537	1.2222	1.4087	1.5791
11		0.2811	0.4560	0.7102	0.9325	1.1961	1.3790	1.5491
12		0.2751	0.4458	0.6929	0.9095	1.1699	1.3511	1.5222
13		0.2683	0.4351	0.6771	0.8895	1.1439	1.3249	1.4940
14		0.2627	0.4261	0.6628	0.8696	1.1191	1.2960	1.4663
15		0.2573	0.4169	0.6483	0.8514	1.0956	1.2705	1.4389
16		0.2520	0.4083	0.6352	0.8336	1.0731	1.2457	1.4108
17		0.2470	0.3998	0.6219	0.8175	1.0523	1.2206	1.3842
18		0.2423	0.3924	0.6101	0.8013	1.0316	1.1979	1.3581
19		0.2378	0.3854	0.5989	0.7869	1.0136	1.1753	1.3320
20		0.2336	0.3784	0.5880	0.7721	0.9937	1.1537	1.3083
21		0.2296	0.3717	0.5777	0.7581	0.9758	1.1317	1.2842
22		0.2256	0.3657	0.5680	0.7450	0.9602	1.1140	1.2621
23		0.2221	0.3601	0.5587	0.7329	0.9432	1.0932	1.2404
24		0.2189	0.3544	0.5501	0.7219	0.9285	1.0765	1.2210
25		0.2154	0.3491	0.5416	0.7107	0.9139	1.0603	1.2019
26		0.2126	0.3440	0.5335	0.6993	0.8994	1.0433	1.1826
27		0.2094	0.3392	0.5260	0.6891	0.8856	1.0269	1.1638
28		0.2070	0.3347	0.5187	0.6793	0.8727	1.0123	1.1472
29		0.2041	0.3302	0.5118	0.6701	0.8609	0.9985	1.1300
30		0.2013	0.3256	0.5043	0.6610	0.8496	0.9834	1.1137
31		0.1989	0.3217	0.4980	0.6516	0.8365	0.9693	1.0988
32		0.1968	0.3181	0.4922	0.6437	0.8267	0.9564	1.0821
33		0.1942	0.3142	0.4858	0.6356	0.8152	0.9435	1.0684
34		0.1919	0.3104	0.4801	0.6274	0.8054	0.9317	1.0550
35		0.1897	0.3066	0.4741	0.6200	0.7953	0.9207	1.0406
36		0.1875	0.3032	0.4690	0.6133	0.7863	0.9087	1.0283
37		0.1855	0.3000	0.4644	0.6064	0.7768	0.8990	1.0178
38		0.1837	0.2969	0.4592	0.5998	0.7686	0.8888	1.0046
39		0.1819	0.2937	0.4539	0.5932	0.7606	0.8794	0.9946
40		0.1799	0.2905	0.4493	0.5868	0.7521	0.8685	0.9825
41		0.1781	0.2876	0.4443	0.5804	0.7432	0.8582	0.9703
42		0.1762	0.2848	0.4403	0.5749	0.7359	0.8502	0.9611
43		0.1746	0.2819	0.4355	0.5690	0.7286	0.8417	0.9516
44		0.1731	0.2794	0.4315	0.5631	0.7207	0.8333	0.9417
45		0.1717	0.2770	0.4277	0.5584	0.7136	0.8242	0.9305
46		0.1697	0.2742	0.4237	0.5526	0.7063	0.8161	0.9218
47		0.1681	0.2717	0.4196	0.5473	0.7001	0.8080	0.9121
48		0.1671	0.2695	0.4161	0.5424	0.6933	0.8008	0.9044
49		0.1655	0.2671	0.4124	0.5378	0.6870	0.7929	0.8959
50		0.1642	0.2649	0.4086	0.5328	0.6810	0.7853	0.8860
51		0.1627	0.2625	0.4051	0.5280	0.6748	0.7787	0.8782
52		0.1613	0.2605	0.4017	0.5239	0.6687	0.7715	0.8708
53		0.1599	0.2581	0.3985	0.5191	0.6626	0.7647	0.8633

continues

Table A13 (continued). Critical values for skewness discordancy test N14 of an extreme outlier in a normal sample.

n	CL SL α	70% 30% 0.30	80% 20% 0.20	90% 10% 0.10	***95%*** ***5%*** ***0.05***	98% 2% 0.02	***99%*** ***1%*** ***0.01***	99.5% 0.5% 0.005
54		0.1587	0.2564	0.3953	0.5155	0.6576	0.7582	0.8559
55		0.1577	0.2544	0.3921	0.5104	0.6520	0.7520	0.8484
56		0.1564	0.2523	0.3893	0.5071	0.6466	0.7452	0.8400
57		0.1553	0.2507	0.3861	0.5028	0.6414	0.7392	0.8330
58		0.1540	0.2487	0.3835	0.4991	0.6359	0.7326	0.8252
59		0.1530	0.2468	0.3806	0.4948	0.6312	0.7268	0.8198
60		0.1518	0.2452	0.3776	0.4917	0.6268	0.7219	0.8129
61		0.1508	0.2433	0.3749	0.4880	0.6218	0.7160	0.8064
62		0.1498	0.2417	0.3723	0.4843	0.6172	0.7104	0.7989
63		0.1488	0.2401	0.3697	0.4811	0.6123	0.7052	0.7933
64		0.1476	0.2383	0.3671	0.4775	0.6082	0.7000	0.7876
65		0.1467	0.2368	0.3647	0.4743	0.6039	0.6949	0.7824
66		0.1459	0.2352	0.3620	0.4709	0.5994	0.6897	0.7767
67		0.1448	0.2336	0.3597	0.4679	0.5954	0.6846	0.7713
68		0.1439	0.2321	0.3573	0.4643	0.5909	0.6799	0.7655
69		0.1430	0.2306	0.3549	0.4615	0.5875	0.6755	0.7598
70		0.1421	0.2293	0.3525	0.4584	0.5828	0.6696	0.7538
71		0.1412	0.2277	0.3505	0.4554	0.5793	0.6661	0.7493
72		0.1403	0.2262	0.3483	0.4527	0.5754	0.6619	0.7454
73		0.1395	0.2249	0.3463	0.4496	0.5719	0.6566	0.7389
74		0.1387	0.2236	0.3441	0.4471	0.5685	0.6537	0.7354
75		0.1379	0.2224	0.3422	0.4444	0.5648	0.6493	0.7304
76		0.1372	0.2209	0.3398	0.4415	0.5614	0.6446	0.7251
77		0.1364	0.2197	0.3379	0.4391	0.5580	0.6404	0.7203
78		0.1355	0.2185	0.3361	0.4363	0.5547	0.6367	0.7156
79		0.1346	0.2170	0.3344	0.4341	0.5512	0.6337	0.7126
80		0.1340	0.2160	0.3325	0.4318	0.5482	0.6298	0.7075
81		0.1333	0.2150	0.3306	0.4294	0.5443	0.6258	0.7029
82		0.1325	0.2138	0.3287	0.4266	0.5415	0.6222	0.6990
83		0.1317	0.2125	0.3272	0.4244	0.5391	0.6186	0.6949
84		0.1311	0.2114	0.3251	0.4219	0.5359	0.6157	0.6920
85		0.1304	0.2104	0.3235	0.4197	0.5326	0.6118	0.6866
86		0.1298	0.2091	0.3215	0.4172	0.5296	0.6082	0.6839
87		0.1291	0.2081	0.3199	0.4153	0.5268	0.6051	0.6799
88		0.1285	0.2071	0.3182	0.4130	0.5240	0.6017	0.6761
89		0.1278	0.2060	0.3166	0.4106	0.5211	0.5985	0.6718
90		0.1273	0.2051	0.3150	0.4090	0.5188	0.5953	0.6679
91		0.1266	0.2040	0.3134	0.4065	0.5158	0.5922	0.6646
92		0.1259	0.2030	0.3119	0.4048	0.5133	0.5888	0.6617
93		0.1254	0.2020	0.3104	0.4026	0.5108	0.5856	0.6568
94		0.1249	0.2010	0.3087	0.4006	0.5077	0.5831	0.6540
95		0.1240	0.2000	0.3075	0.3987	0.5056	0.5800	0.6516
96		0.1237	0.1991	0.3059	0.3966	0.5030	0.5776	0.6480
97		0.1230	0.1979	0.3044	0.3949	0.5005	0.5742	0.6434
98		0.1223	0.1972	0.3028	0.3928	0.4980	0.5711	0.6409
99		0.1219	0.1963	0.3016	0.3910	0.4956	0.5686	0.6380
100		0.1213	0.1956	0.3001	0.3890	0.4931	0.5653	0.6344

CL: Confidence level (%); SL: Significance level (%); α: Significance level. The headers for CL or SL or α generally used for most applications are given in italic-bold face. The mean values of the standard error of the mean ($\bar{x}_{se}$) for these critical values ($\bar{x}$) are (respective % errors are also reported in parentheses): ~0.00020 (for α=0.30, 0.11%); ~0.00021 (for α=0.20, 0.07%); ~0.00024 (for α=0.10, 0.05%); ~0.00032 (for α=0.05, 0.05%); ~0.00048 (for α=0.02, 0.06%); ~0.0006 (for α=0.01, 0.08%); and ~0.0009 (for α=0.005, 0.10%).

Table A14. Critical values for kurtosis discordancy test N15 of an extreme outlier in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
3		---	---	---	---	---	---	---
4		---	---	---	---	---	---	---
5		2.2998	2.4668	2.6928	2.8764	3.0381	3.1141	3.1636
6		2.4191	2.6661	2.9933	3.2798	3.5827	3.7512	3.8763
7		2.4956	2.7782	3.1920	3.5466	3.9632	4.2215	4.4315
8		2.5586	2.8545	3.3166	3.7305	4.2301	4.5640	4.8515
9		2.6166	2.9128	3.4013	3.8560	4.4136	4.8064	5.1606
10		2.6710	2.9624	3.4607	3.9421	4.5464	4.9830	5.3865
11		2.7161	3.0067	3.5058	4.0034	4.6441	5.1066	5.5518
12		2.7534	3.0448	3.5421	4.0457	4.7052	5.1958	5.6674
13		2.7853	3.0762	3.5732	4.0797	4.7588	5.2665	5.7651
14		2.8123	3.1026	3.5964	4.1024	4.7845	5.3053	5.8197
15		2.8359	3.1243	3.6150	4.1186	4.8073	5.3351	5.8620
16		2.8569	3.1430	3.6303	4.1306	4.8179	5.3536	5.8924
17		2.8759	3.1594	3.6440	4.1423	4.8260	5.3605	5.9078
18		2.8921	3.1734	3.6528	4.1472	4.8276	5.3629	5.9184
19		2.9071	3.1860	3.6612	4.1504	4.8279	5.3600	5.9101
20		2.9206	3.1966	3.6669	4.1519	4.8232	5.3534	5.9015
21		2.9323	3.2058	3.6710	4.1504	4.8150	5.3476	5.8983
22		2.9435	3.2146	3.6752	4.1501	4.8091	5.3313	5.8731
23		2.9532	3.2224	3.6788	4.1472	4.8019	5.3216	5.8653
24		2.9628	3.2285	3.6797	4.1431	4.7879	5.3058	5.8492
25		2.9710	3.2343	3.6815	4.1398	4.7809	5.2909	5.8257
26		2.9784	3.2390	3.6804	4.1361	4.7688	5.2748	5.8035
27		2.9868	3.2452	3.6809	4.1295	4.7563	5.2562	5.7844
28		2.9924	3.2482	3.6794	4.1227	4.7424	5.2362	5.7592
29		2.9989	3.2519	3.6793	4.1180	4.7302	5.2178	5.7347
30		3.0042	3.2546	3.6781	4.1110	4.7167	5.2029	5.7168
31		3.0097	3.2580	3.6767	4.1056	4.7057	5.1853	5.6913
32		3.0150	3.2609	3.6741	4.0982	4.6892	5.1634	5.6645
33		3.0193	3.2633	3.6734	4.0939	4.6773	5.1451	5.6415
34		3.0241	3.2655	3.6712	4.0860	4.6648	5.1287	5.6177
35		3.0278	3.2672	3.6693	4.0799	4.6540	5.1136	5.5992
36		3.0316	3.2687	3.6671	4.0725	4.6371	5.0924	5.5733
37		3.0351	3.2704	3.6653	4.0668	4.6257	5.0744	5.5495
38		3.0383	3.2716	3.6627	4.0604	4.6130	5.0579	5.5341
39		3.0414	3.2733	3.6596	4.0530	4.6013	5.0421	5.5073
40		3.0444	3.2742	3.6580	4.0468	4.5899	5.0264	5.4925
41		3.0472	3.2748	3.6545	4.0400	4.5768	5.0094	5.4678
42		3.0497	3.2757	3.6523	4.0333	4.5618	4.9878	5.4448
43		3.0524	3.2763	3.6493	4.0274	4.5534	4.9786	5.4270
44		3.0550	3.2776	3.6463	4.0209	4.5422	4.9621	5.4068
45		3.0568	3.2776	3.6443	4.0148	4.5295	4.9422	5.3843
46		3.0592	3.2782	3.6421	4.0090	4.5188	4.9285	5.3619
47		3.0609	3.2780	3.6389	4.0038	4.5090	4.9152	5.3458
48		3.0630	3.2788	3.6361	3.9981	4.4959	4.8974	5.3259
49		3.0647	3.2788	3.6336	3.9913	4.4862	4.8846	5.3060
50		3.0661	3.2785	3.6305	3.9850	4.4728	4.8673	5.2867
51		3.0683	3.2790	3.6277	3.9777	4.4678	4.8563	5.2670
52		3.0694	3.2792	3.6253	3.9743	4.4551	4.8401	5.2510
53		3.0711	3.2790	3.6224	3.9675	4.4450	4.8281	5.2352

continues

Table A14 (continued). Critical values for kurtosis discordancy test N15 of an extreme outlier in a normal sample.

n	CL	70%	80%	90%	***95%***	98%	***99%***	99.5%
	SL	30%	20%	10%	***5%***	2%	***1%***	0.5%
	α	0.30	0.20	0.10	***0.05***	0.02	***0.01***	0.005
54		3.0727	3.2790	3.6196	3.9629	4.4360	4.8167	5.2162
55		3.0736	3.2786	3.6173	3.9567	4.4241	4.7986	5.1966
56		3.0749	3.2787	3.6149	3.9505	4.4138	4.7860	5.1807
57		3.0764	3.2787	3.6121	3.9455	4.4064	4.7743	5.1649
58		3.0779	3.2786	3.6084	3.9397	4.3947	4.7610	5.1461
59		3.0784	3.2782	3.6063	3.9347	4.3869	4.7489	5.1304
60		3.0796	3.2784	3.6041	3.9296	4.3781	4.7365	5.1177
61		3.0807	3.2777	3.6014	3.9253	4.3701	4.7256	5.1013
62		3.0818	3.2777	3.5989	3.9205	4.3605	4.7117	5.0854
63		3.0825	3.2770	3.5959	3.9140	4.3514	4.7027	5.0702
64		3.0838	3.2774	3.5939	3.9086	4.3430	4.6897	5.0519
65		3.0841	3.2763	3.5909	3.9050	4.3353	4.6772	5.0393
66		3.0852	3.2769	3.5886	3.9006	4.3279	4.6683	5.0243
67		3.0858	3.2761	3.5868	3.8961	4.3200	4.6568	5.0120
68		3.0870	3.2759	3.5836	3.8898	4.3113	4.6485	5.0031
69		3.0876	3.2755	3.5821	3.8861	4.3029	4.6344	4.9832
70		3.0883	3.2749	3.5789	3.8822	4.2962	4.6239	4.9694
71		3.0888	3.2745	3.5773	3.8781	4.2869	4.6137	4.9619
72		3.0892	3.2737	3.5749	3.8733	4.2818	4.6049	4.9474
73		3.0901	3.2734	3.5722	3.8701	4.2757	4.5954	4.9316
74		3.0907	3.2730	3.5700	3.8650	4.2650	4.5841	4.9203
75		3.0908	3.2726	3.5676	3.8601	4.2588	4.5737	4.9079
76		3.0914	3.2722	3.5657	3.8569	4.2522	4.5661	4.8973
77		3.0920	3.2717	3.5632	3.8518	4.2456	4.5563	4.8860
78		3.0928	3.2714	3.5610	3.8473	4.2379	4.5470	4.8721
79		3.0931	3.2707	3.5597	3.8441	4.2316	4.5375	4.8602
80		3.0935	3.2703	3.5569	3.8397	4.2255	4.5309	4.8520
81		3.0941	3.2698	3.5555	3.8370	4.2175	4.5218	4.8415
82		3.0945	3.2694	3.5530	3.8327	4.2123	4.5132	4.8290
83		3.0948	3.2687	3.5509	3.8290	4.2052	4.5039	4.8216
84		3.0949	3.2684	3.5485	3.8244	4.2001	4.4978	4.8110
85		3.0957	3.2680	3.5464	3.8212	4.1944	4.4895	4.7958
86		3.0960	3.2673	3.5444	3.8173	4.1880	4.4815	4.7870
87		3.0962	3.2668	3.5427	3.8141	4.1822	4.4709	4.7783
88		3.0968	3.2663	3.5408	3.8107	4.1770	4.4658	4.7707
89		3.0970	3.2661	3.5395	3.8069	4.1695	4.4559	4.7622
90		3.0973	3.2650	3.5368	3.8034	4.1643	4.4472	4.7485
91		3.0975	3.2651	3.5353	3.8009	4.1591	4.4412	4.7400
92		3.0979	3.2647	3.5334	3.7969	4.1534	4.4337	4.7299
93		3.0981	3.2642	3.5313	3.7944	4.1483	4.4274	4.7229
94		3.0982	3.2635	3.5297	3.7911	4.1426	4.4196	4.7137
95		3.0984	3.2630	3.5278	3.7872	4.1373	4.4132	4.7041
96		3.0990	3.2626	3.5258	3.7828	4.1311	4.4061	4.6959
97		3.0991	3.2618	3.5233	3.7798	4.1260	4.3984	4.6867
98		3.0991	3.2613	3.5220	3.7772	4.1199	4.3911	4.6759
99		3.0991	3.2607	3.5201	3.7747	4.1153	4.3830	4.6659
100		3.0994	3.2605	3.5190	3.7713	4.1113	4.3783	4.6602

CL: Confidence level (%); SL: Significance level (%); α: Significance level. The headers for CL or SL or α generally used for most applications are given in italic-bold face. The mean values of the standard error of the mean ($\bar{x}_{se}$) for these critical values ($\bar{x}$) are (respective % errors are also reported in parentheses): ~0.00025 (for α=0.30, 0.009%); ~0.00033 (for α=0.20, 0.010%); ~0.0005 (for α=0.10, 0.014%); ~0.0007 (for α=0.05, 0.017%); ~0.0011 (for α=0.02, 0.025%); ~0.0017 (for α=0.01, 0.035%); and ~0.0025 (for α=0.005, 0.05%).